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**Question Paper Code: U2M10**

B.E./B.Tech. DEGREE EXAMINATION, NOV 2022

Second Semester

Computer Science and Design

21UMA210- Differential equations, Fourier series & Transforms

(Regulations 2021)

(Common to Artificial Intelligence and Data Science)

Duration: Three hours

Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

- The complete solution of  $(x^2D^2 - 3xD - 5)y = 0$  is \_\_\_\_\_. CO1-App  
(a)  $Ae^{-z} + B e^{5z}$       (b)  $Ae^z + B e^{5z}$       (c)  $Ae^z + B e^{-5z}$       (d)  $Ae^{-z} + B e^{-5z}$
- $\frac{1}{D^2}(\cos x) =$  \_\_\_\_\_. CO6-R  
(a)  $\sin x$       (b)  $-\cos x$       (c)  $\cos x$       (d)  $\tan x$
- $L(\sin h at) =$  \_\_\_\_\_. CO2-App  
(a)  $\frac{s}{s^2 - a^2}$       (b)  $\frac{a}{s^2 - a^2}$       (c)  $\frac{s}{s^2 + a^2}$       (d)  $\frac{a}{s^2 + a^2}$
- $L(te^{-3t})$  CO2-App  
(a)  $-z \frac{d}{dz} F(Z)$       (b)  $z \frac{d}{dz} F(Z)$       (c)  $\frac{d}{dz} F(Z)$       (d)  $z^n \frac{d}{dz} F(Z)$
- $Z[n f(n)] =$  \_\_\_\_\_. CO3- App  
(a)  $-z \frac{d}{dz} F(Z)$       (b)  $z \frac{d}{dz} F(Z)$       (c)  $\frac{d}{dz} F(Z)$       (d)  $z^n \frac{d}{dz} F(Z)$
- The Z transform of  $n2^n$  is \_\_\_\_\_. CO3- App  
(a)  $\frac{2z}{(z-2)^2}$       (b)  $\frac{z}{(z-2)^2}$       (c)  $\frac{2z}{(z+2)^2}$       (d)  $\frac{z}{(z+2)^2}$

7. The root mean square value of  $f(x) = x^2$  in  $(0, 1)$  is -----.
- (a) 1                                      (b)  $1/2$                                       (c)  $1/\sqrt{3}$                                       (d) 2                                      CO4-App
8. The constant term in the Fourier expansion for  $f(x) = k$ ,  $(0, 2\pi)$  is
- (a)  $2k$                                       (b)  $3k$                                       (c)  $k$                                       (d) 0                                      CO4-App
9. Convolution theorem on Fourier Transform is  $F[f(x)*g(x)] =$
- \_\_\_\_\_
- (a)  $F(s).G(s)$                                       (b)  $f(s).g(s)$                                       (c)  $F(s)*G(s)$                                       (d)  $f(s)*g(s)$                                       CO6-U
10.  $F[xf(x)] =$  \_\_\_\_\_
- (a)  $-F_c[f(x)]$                                       (b)  $-\frac{d}{ds}\{F_s[f(x)]\}$                                       (c)  $-F_s[f(x)]$                                       (d)  $-\frac{d}{ds}\{F_c[f(x)]\}$                                       CO5-App

PART – B (5 x 2= 10Marks)

11. Find Particular integral for
- $(D^2 - 2D + 1)y = \cosh x$
- CO1-App
12. Evaluate  $L[\cos at]$
- CO2-App
13. Evaluate  $z\left(\frac{1}{n+1}\right)$
- CO3-App
14. Give the expression for the Fourier series coefficient  $b_n$  for the function  $f(x) = x^2$  defined in  $-2 \leq x \leq 2$ .
- CO4-App
15. Find the Fourier Sine Transform of  $f(x) = e^{-x}, x > 0$
- CO5-App

PART – C (5 x 16= 80Marks)

16. (a) (i) Solve  $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$                                       CO1-App                                      (8)
- (ii) Solve  $(D^2 + 2D + 2)y = e^{-2x} + \cos 2x$                                       CO1- App                                      (8)
- Or
- (b) (i) Solve  $(D^2 - 3D + 2)y = 2e^x + 2\cos 2x$                                       CO1- App                                      (8)
- (ii) Solve  $(3z - 4y)p + (4x - 2z)q = 2y - 3x$                                       CO1- App                                      (8)
17. (a) Find the Laplace transform of  $f(t) = \begin{cases} t, & 0 < t < a \\ 2a - t, & a < t < 2a \end{cases}$  and  $f(t + 2a) = f(t)$                                       CO2-App                                      (16)

Or

- (b) Solve by the convolution theorem  $L^{-1}\left[\frac{s^2}{(s^2 + 5^2)(s^2 + 4^2)}\right]$  CO2 -App (16)

Using Laplace Transforms

18. (a) Using convolution theorem find the inverse Z-transform  $Z^{-1}\left(\frac{8z^2}{(2z-1)(4z-1)}\right)$  CO3-App (16)

Or

- (b) (i) Evaluate  $Z[a^n \cos n\theta]$  and  $Z[a^n \sin n\theta]$  CO3-App (8)

(ii) Evaluate

CO3-App (8)

$$Z^{-1}\left[\frac{z}{z^2 + 4z + 3}\right]$$

19. (a) Find the Fourier series of  $f(x) = x^2$  in  $(-\pi, \pi)$  of periodicity  $2\pi$ . CO4-App (16)

Or

- (b) (i) The table of values of the function  $y = f(x)$  is given below: CO4-App (8)

x:	0	1	2	3	4	5
Y:	4	8	15	7	6	2

Find a Fourier series up to the third harmonic to represent  $f(x)$  in terms of  $x$ .

- (ii) Find the half range sine series for  $f(x) = x^2$  in  $(0, l)$  CO4-App (8)

20. (a) Show that the Fourier transform of CO5-App (16)

$$f(x) = \begin{cases} a^2 - x^2 & |x| < a \\ 0 & |x| > a > 0 \end{cases} \quad \text{is } 2\sqrt{\frac{2}{\pi}} \left[ \frac{\sin sa - sa \cos sa}{s^3} \right] \quad \text{Hence}$$

$$\text{deduce } \int_0^{\infty} \frac{\sin t - t \cos t}{t^3} dt = \pi/4 \quad \text{and} \quad \int_0^{\infty} \left( \frac{\sin t - t \cos t}{t^3} \right)^2 dt = \pi/15$$

Or

(b) (i) Find the Fourier sine & cosine transform of  $e^{-ax}$

CO5-App (8)

(ii) Evaluate  $\int_0^{\infty} \frac{dx}{(x^2 + 25)^2}$

CO5-App (8)