A **Reg. No. :** | | | | | | | | | | |

Question Paper Code: 54024

B.E. / B.Tech. DEGREE EXAMINATION, DEC 2021

Fourth Semester

Electronics and Communication Engineering

15UMA424 - PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering)

(Regulation 2015)

(Statistical tables may be permitted)

Duration: Three hours Maximum: 100 Marks

PART A - (10 x 1 = 10 Marks)

1. When X and Y are independent random variables $M_{X+Y}(t)$ = CO1-R

(a) M_X (t) M_Y(t) (b) M_{XY}(t) (c) M_{YX} (t) (d) M_X (t) + M_Y(t)

2. If the moment generating function of a binomial random variable X is of the form $(0.4e^t + 0.6)^8$, then its mean is CO1-R

(a) 16 (b) 16/5 (c) 16/3 (d) 14/16

3. If the joint probability density function of a bivariate random variable (X,Y) is f(x,y) = k , 0 < x< 1, 0 < y < 1, then the value of k is CO2-R

(a) 1 (b) 4 (c) 2 (d) 3

4. When X and Y are uncorrelated random variables , the covariance of X and Y is i.e., cov(x,y) = CO2-R

(a) 1 (b) -1 (c) 0 (d) 0.5

5. If both parameter set T and state space S are discrete, then the random process is known as CO3-R

(a) discrete random sequence (b) continuous random process

(c) discrete random process (d) continuous random sequence

6. Sum of two independent Poisson processes is a CO3-R

(a) Gaussian process (b) Poisson process (c) Ergodic process (d) Binomial process

7. Auto correlation function is an CO4-R

(a) odd function (b) complex function (c) invalid function (d) even function

8. If $\{X(t)\}$ and $\{Y(t)\}$ are two random processes then $|R_{XY}(\tau)| \leq$ CO4-R

(a) $\sqrt{R_{XX}(0)R_{YY}(0)}$ (b) $R_{XX}(0) + R_{YY}(0)$ (c) $R_{XX}(0)/R_{YY}(0)$ (d) 0

9. The convolution form of the output Y(t) of a linear time invariant system with the input X(t) and the system weighting function h(t) is CO5-R

(a) $\int_{-\infty}^{\infty} h(u)\ du$ (b) $\int_{-\infty}^{\infty} h(u)\, X(t - u\, du$ (c) $\int_{-\infty}^{\infty} h(u)\, y(t-u)\, du$ (d) $\int_{-\infty}^{\infty} X(t - u\, du$

10. When the auto correlation function of the random telegraph signal process is $R(\tau) = a^2 e^{-2\gamma|\tau|}$ then its power spectral density is given by CO5-R

(a) $\frac{4a^2\gamma}{4\gamma^2 + \omega^2}$ (b) $2\,\delta(\tau)$ (c) $4a^2\gamma$ (d) $\delta(\tau)$

PART – B (5 x 2= 10Marks)

11. If $(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{3}$ and $P(\bar{B}) = \frac{1}{2}$, find $P(A)$ and $P(B)$. CO1-R

12. Find E(XY) using the joint probability density function $f(x, y) = \begin{cases} 8xy, & 0 \leq x \leq 1 ; 0 \leq y \leq x \\ 0, & else\ where \end{cases}$ CO2-R

13. Define Wide sense stationary process. CO3-R

14. Define the Power spectral density. CO4-R

15. Define the system function or power transfer function. CO5-R

PART – C (5 x 16= 80Marks)

16. (a) (i) A student buys 1000 integrated circuits (ICs) from supplier A, 2000 ICs from supplier B, and 3000 ICs from supplier C. He tested the ICs and found that the probability of getting a defective IC given that it came from supplier A is 0.05, probability of getting a defective IC given that it came from supplier B is 0.10 and probability of getting a defective IC given that it came from supplier C is 0.10. If the ICs from the three suppliers are mixed together and one is selected at random, what is the probability that it is defective? CO1 -App (8)

(ii) A random variable 'X' has the following probability function CO1 -App (8)

Values of X	0	1	2	3	4	5	6	7	8
Probability P[X = x]	a	3a	5a	7a	9a	11a	13a	15a	17a

1) Determine the value of 'a'.
2) Find P[X ≥ 3]
3) Find P[0 < X < 5].

Find the distribution function of X.

Or

(b) (i) Obtain the Moment Generating Function of Binomial distribution and hence find its mean and variance — CO1 -App (8)

(ii) The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = 1/2$. — CO1 -App (8)

(1) What is the probability that the repair time exceeds 2 hours?

(2) What is the conditional probability that a repair takes at 11 hours given that its direction exceeds 8 hours?

17. (a) (i) The two dimensional random variable (X,Y) has the joint density function $f(x,y) = x + 2y$, $x = 0,1,2$; $y = 0,1,2$ — CO2 -App (8)

(1) Find the value of k.

(2) Find the marginal distribution of X and Y.

(3) Find the conditional distribution of Y for X=x.

(ii) If X and Y each follow an exponential distribution with parameter 1 and are independent, find the probability density function of U = X – Y. — CO2 -App (8)

Or

(b) Two random variables X and Y have the following joint probability density function — CO2- Ana (16)

$$f(x,y) = \begin{cases} 2 - x - y, & 0 \le x \le 1 , 0 \le y \le 1 \\ 0, & otherwise \end{cases}$$

Find the correlation coefficient of (X,Y).

18. (a) (i) Show that the random process $X(t) = Kcos\,(\omega t + \theta)$ is wide sense stationary if K & ω are constant and 'θ' is uniformly distributed random variable in (0, 2π). — CO3- Ana (8)

(ii) If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between 2 consecutive arrivals is (1) more than 1 minute (2) between 1 minute and 2 minutes and (3) 4 minutes or less. — CO3- Ana (8)

Or

	(b)	(i) A salesman territory consists of three cities A,B and C. He never sells in the same city on successive days. If he sells in A, then the next day he sells in city B. However if he sells in either B or C, the next day he is twice as likely to sell in the city A as in the other city. In the long run, how often does he sell in each of the cities?	CO3- Ana	(8)
		(ii) Consider the process $X(t) = A\cos t + B\sin t$, where A and B are uncorrelated random variables each with mean 0 and variance 2 .Show that the process X(t) is covariance stationary.	CO3- Ana	(8)
19.	(a)	(i) Two random processes X(t) and Y(t) are defined as follows: $X(t) = A\,COS(\omega t + \Theta)$; $Y(t) = B\,Sin(\omega t + \Theta)$ where A, B and ω are constants and Θ is a random variable that is uniformly distributed between 0 and 2π.Find the cross correlation function of X(t) and Y(t).	CO4- App	(8)
		(ii) Find the power spectral density of a WSS process with autocorrelation function $R(\tau) = \sigma^2\, cosp\tau$.	CO4- App	(8)
		Or		
	(b)	(i) The power spectral density function of a zero mean WSS process {X(t)} is given by $S(\omega) = \begin{cases} 1 & \lvert\omega\rvert < \omega_0 \\ 0 & else\ where \end{cases}$. Find $R(\tau)$ and show also that $X(t)$ and $X\left(t + \frac{\tau}{\omega_0}\right)$ are uncorrelated.	CO4- Ana	(8)
		(ii) If $R_{yy}(\tau) = 2R_{xx}(\tau) - R_{xx}(\tau + 2a) - R_{xx}(\tau - 2a)$, prove that $S_{yy}(\omega) = 4sin^2 a\omega S_{xx}(\omega)$.	CO4- Ana	(8)
20.	(a)	(i) Let Y(t) = X(t) + N(t) be a wide-sense stationary process where X(t) is the actual signal and N(t) is a zero-mean noise process with variance σ_N^2 and independent of X(t). Find the power spectral density of Y(t).	CO5 -U	(8)
		(ii) If $\{X(t)\}$ is a WSS process and if $Y(t) = \int_{-\infty}^{\infty} h(u)\, X(t - u\, du$, then prove that the system is a linear time-invariant system.	CO5- U	(8)
		Or		
	(b)	A Wide sense stationary process X(t) is the input to a linear system whose impulse response is $h(t) = 2e^{-7t}; t \geq 0$. The autocorrelation of the function of the process is $R_{XX}(\tau) = e^{-4\lvert\tau\rvert}$.Find the power spectral density of the output process Y(t).	CO5 -App	(16)