Reg. No. : | | | | | | | | | | |

Question Paper Code: 41044

B.E. / B.Tech. DEGREE EXAMINATION, DEC 2021

Fourth Semester

Electronics and Communication Engineering

14UMA424 - PROBABILITY AND RANDOM PROCESS

(Regulation 2014)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

(Statistical Tables are permitted)

PART A - (10 x 1 = 10 Marks)

1. The probability of impossible event is
   (a) 1 (b) 0 (c) 2 (d) 0.5

2. In which probability distribution, Variance and Mean are equal?
   (a) Binomial (b) Poisson (c) Geometric (d) None of these

3. If two random variables X and Y are independent, then covariance is
   (a) θ (b) 1 (c) 0 (d) λ

4. If $X=Y$ then correlation coefficient between them is
   (a) *0* (b) ∞ (c) *1* (d) ± 1

5. The sum of two independent Poisson process is
   (a) poisson process (b) marcov process
   (c) random process (d) stationary

6. A Non-Null Persistent and Aperiodic state is called
   (a) Return state (b) Irreducible (c) Ergodic (d) Recurrent

7. $R_{XX}(\tau)$ is an _________ function of τ
   (a) positive (b) 1 (c) even (d) odd

8. If $R_{xy}(\tau) = \mu_X \times \mu_Y$ then *X(t)* and *Y(t)* are called
   (a) Independent (b) Orthogonal (c) Stationary (d) none of these

9. A __________ is a functional relationship between the input *X (t)* and the output *Y (t)*
   (a) system (b) process (c) functional (d) stationary

10. Colouted Noise means a noise that is
   (a) white (b) not white (c) coloured (d) none of these

PART - B (5 x 2 = 10 Marks)

11. If a Random variable X has the moment generating function $M_x(t)=\frac{2}{2-t}$. Determine the variance of X.

12. Define covariance.

13. Outline discrete random process. Give an example for it.

14. State Winear-Khinchine theorem.

15. Describe a linear system.

PART - C (5 x 16 = 80 Marks)

16. (a) (i) In a bolt factory machines *A, B, C* manufacture respectively 25%, 35% and 40% of the total. Of their total output 5%, 4% and 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines *A, B* and *C*? (8)

(ii) Deduce the moment generating function of exponential distribution and hence find its mean and variance. (8)

Or

(b) A random variable X has the following probability function

Value of x	0	1	2	3	4
$P(x)$	k	$3k$	$5k$	$7k$	$9k$

Find the value of k, $P(x < 3)$ and distribution function of x. (16)

17. (a) (i) The joint probability density function of a bivariate random variable (X, Y) is $f(x,y) = \begin{cases} k(x+y), 0 < x < 2, 0 < y < 2 \\ 0 \quad , elsewhere \end{cases}$. Find (1) the value of k (2) the marginal probability density of x and y (3) x and y independent. (8)

(ii) The two lines of regression are $8x - 10y + 66 = 0$, $40x - 18y - 214 = 0$. The variance of x is 9. Evaluate the mean values of x and y and the Correlation coefficient between x and y. (8)

Or

(b) The joint probability distribution of X and Y is given below:

Y \ X	-1	1
0	1/8	3/8
1	2/8	2/8

Find the correlation coefficient between X and Y. (16)

18. (a) Generalize the postulates of a Poisson process and derive the probability distribution for the Poisson Process. Also Show that the sum of two independent Poisson process is again a Poisson process. (16)

Or

(b) (i) Explain the classification of random process. (8)

(ii) The transition probability of a Markov chain $\{X\}$, $n = 1, 2, 3\ldots\ldots$, having 3 states 1, 2 and 3 $P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$ and the initial distribution is $p^{(0)} = (0.7, 0.2, 0.1)$.

Find (1) $P\{X_2 = 3\}$ and (2) $P = \{X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2\}$. (8)

19. (a) (i) Define cross-correlation function and write the properties of cross-correlation function. (8)

(ii) State and Prove Wiener-Khinchine theorem. (8)

Or

(b) If the power spectral density of a WSS process is given by

$$S(\omega) = \begin{cases} \frac{b}{a}(a - |\omega|), & for\ |\omega| \le a \\ 0, & for\ |\omega| > a \end{cases}$$

Find the autocorrelation function of the process. (16)

20. (a) If $\{X(t)\}$ is a WSS process and if $y(t) = \int_{-\infty}^{\infty} h(u)X(t-u)du$ then Prove that

(i) $R_{XY}(\tau) = R_{XX}(\tau) * h(-\tau)$ (ii) $R_{YY}(\tau) = R_{XY}(\tau) * h(\tau)$

(iii) $S_{XY}(\omega) = S_{XX}(\omega)H^*(\omega)$ (iv) $S_{YY}(\omega) = S_{XX}(\omega)|H(\omega)|^2$ (16)

Or

(b) (i) The input to a time- invariant, stable linear system is a WSS process, Enumerate that the output will also be a WSS process. (8)

(ii) If *X(t)* is a band limited process such that $S_{xx}(\omega) = 0, |\omega| > \sigma$, then formulate $2[R_{xx}(0) - R_{xx}(\tau)] \leq \sigma^2\tau^2 R_{xx}(0)$. (8)
