Reg. No. : | | | | | | | | | | |

Question Paper Code: 41002

B.E. / B.Tech. DEGREE EXAMINATION, DEC 2021

First Semester

Civil Engineering

14UMA102 - ENGINEERING MATHEMATICS – I

(Common to ALL branches)

(Regulation 2014)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions.

PART A - (10 x 1 = 10 Marks)

1. If $A = \begin{pmatrix} 2 & 1 & 4 \\ 0 & 3 & 6 \\ 0 & 0 & 5 \end{pmatrix}$ then eigen values of A^{-1} are

   (a) 2, 3, 5 (b) 2, 1, 4 (c) $\frac{1}{2}, \frac{1}{3}, \frac{1}{5}$ (d) $\frac{1}{2}, 1, \frac{1}{4}$

2. If 0, 3, 4 are eigen values of a square matrix A of order 3 then $|A|$ =

   (a) 12 (b) 0 (c) ∞ (d) $\frac{1}{12}$

3. The harmonic series $\sum \frac{1}{n^p}$ is convergent if

   (a) $p > 1$ (b) $p < 1$ (c) $p = 1$ (d) $p \le 1$

4. D'Alembert's test is also called

   (a) Ratio test (b) Root test (c) Abel's test (d) none of these

5. The radius of curvature of the curve $y = e^x$ at (0,1) is

(a) $2\sqrt{2}$ (b) $\sqrt{2}$ (c) 2 (d) $2\sqrt{3}$

6. The envelope of the family of lines $y = x + a/m$, m being a positive integer

(a) $y^2 = 4ax$ (b) $x^2 = 4ay$ (c) $x^2 + y^2 = a^2$ (d) $xy = a^2$

7. Let u and v be functions of x, y and $u = e^v$. Then u and v are

(a) Functionally dependent (b) Functionally independent
(c) Functionally linear (d) Functionally non-linear

8. A stationary point of $f(x, y)$ at which $f(x, y)$ has neither a maximum nor a minimum is called

(a) Extreme point (b) Max-Min point
(c) Saddle point (d) Nothing can be said

9. The value of $\int_0^a \int_0^{\sqrt{a^2 - x^2}} dy\, dx$ is

(a) $\frac{\pi a^2}{4}$ (b) $\frac{\pi a^2}{2}$ (c) $\frac{\pi a^2}{8}$ (d) πa^2

10. The value of the double integral $\int_0^{\pi} \int_0^{a} r\, dr\, d\theta$ is

(a) πa^2 (b) $\frac{\pi a^2}{2}$ (c) $\frac{\pi r^2}{2}$ (d) πr^2

PART - B (5 x 2 = 10 Marks)

11. Determine the nature of the quadratic form $x_1^2 + 3x_2^2 + 6x_3^2 + 2x_1x_2 + 2x_2x_3 + 4x_3x_1$ without reducing to canonical form.

12. State Leibnitz's test.

13. Find the radius of curvature at the point (c, c) on the curve xy = c^2.

14. If x= $u^2 - v^2$ and y= $2uv$, find the Jacobian of x and y with respect to u and v.

15. Indicate the region of integration of $\int_0^a \int_{\frac{x^2}{a}}^{x} xdydx$.

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PART - C (5 x 16 = 80 Marks)

16. (a) (i) Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$. (8)

(ii) Using Cayley Hamilton theorem,find the inverse of the matrix $\begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{pmatrix}$. (8)

Or

(b) Reduce the Q.F $x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$ in to a canonical form by an orthogonal transformation. (16)

17. (a) (i) Show that the sum of the series $\frac{15}{16} + \frac{15}{16} \times \frac{21}{24} + \frac{15}{16} \times \frac{21}{24} \times \frac{27}{32} + \cdots \infty = \frac{47}{9}$. (8)

(ii) Show that the series $1 - 2 + 3 - 4 + \ldots \infty$ oscillates infinitely. (8)

Or

(b) Prove that if $b - 1 > a > 0$, the series $1 + \frac{a}{b} + \frac{a(a+1)}{b(b+1)} + \frac{a(a+1)(a+2)}{b(b+1)(b+2)} + \ldots\ldots$ converges. (16)

18. (a) (i) Find the radius of curvature at any point of the cycloid

$x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$. (8)

(ii) Find the evolute of the parabola $y^2 = 4ax$. (8)

Or

(b) Considering the evolute as the envelope of normals, find the evolute of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (16)

19. (a) Given the transformations $u = e^x \cos y$ and $v = e^x \sin y$ and that ϕ is a function of u and v and also of x and y, prove that $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = (u^2 + v^2)\left[\frac{\partial^2 \phi}{\partial u^2} + \frac{\partial^2 \phi}{\partial v^2}\right]$. (16)

Or

(b) (i) If $g(x, y) = \psi(u, v)$, where $u = x^2 - y^2$, $v = 2xy$, then prove that

$$\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} = 4(x^2 + y^2)\left(\frac{\partial^2 \psi}{\partial u^2} + \frac{\partial^2 \psi}{\partial v^2}\right).$$ (8)

(ii) Find the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 where $y_1 = \frac{x_2 x_3}{x_1}$, $y_2 = \frac{x_1 x_3}{x_2}$,

$y_3 = \frac{x_1 x_2}{x_3}$. (8)

20. (a) Change the order of integration and hence evaluate it $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy\,dy\,dx$. (16)

Or

(b) (i) Evaluate $\iint_S z^3 dS$, where is S is the positive octant of the surface of the sphere. (8)

(ii) Evaluate $\iiint_V xyz\,dx\,dy\,dz$, where V is the volume of space inside the tetrahedron bounded by the planes x = 0, y = 0, z = 0 and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. (8)
