Reg. No. : | | | | | | | | | | |

Question Paper Code: 32002

B.E. / B.Tech. DEGREE EXAMINATION, DEC 2021

Second Semester

Civil Engineering

01UMA202 - ENGINEERING MATHEMATICS - II

(Common to all branches)

(Regulation 2013)

Duration: Three hours | Maximum: 100 Marks

Answer ALL Questions.

PART A - (10 x 2 = 20 Marks)

1. Find the particular integral of $(D^2 + 4)y = \pi$.
2. Transform $[(2x+3)^2 D^2 -2(2x+3)D -12]\, y = 0$ into an ordinary differential equation.
3. Evaluate $\int_c (2x - y)dx + (x + y)dy$ where c is the boundary of the circle $x^2 + y^2 = 1$ in the XOY plane.
4. If $\vec{F} = axi + by\vec{j} + cz\vec{k}$, then find the value of the integral $\iint_s \vec{F}.\hat{n}\, ds$.
5. Check whether xy^2 is real part of an analytic function.
6. Find the fixed points of $w = \frac{3z - 4}{z - 1}$.
7. Find the residue of $f(z) = \frac{z^2}{(z-1)^2(z-2)}$ $at\ z = 2$.
8. Expand $\frac{1}{z - 2}$ at $z = 1$ in a Taylor's series.

9. State and prove the shifting property in Laplace Transform.

10. Define singular point.

PART - B (5 x 16 = 80 Marks)

11. (a) (i) Solve $(D^2 + 4)y = \cot 2x$ by method of variation of parameters (8)

(ii) Solve $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos[\log(1+x)]$. (8)

Or

(b) (i) Solve $(D^2+2D+5)\, y = e^{-x} \tan x$ by method of variation of parameter. (8)

(ii) Solve $\frac{dx}{dt} + y = \sin t$ and $\frac{dy}{dt} + x = \cos t$ given x = 2, y =0 when t = 0. (8)

12. (a) (i) Find the directional derivative of $\phi = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of $\vec{i} + 2\vec{j} + 2\vec{k}$. (8)

(ii) Prove that $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational vector and find the scalar potential such that $\vec{F} = \Delta\phi$. (8)

Or

(b) Verify Gauss divergence theorem for $\vec{F} = xz\,\vec{i} + 4xy\,\vec{j} - z^2\vec{k}$ over the cube bounded by $x = 0, x = 2, y = 0, y = 2, z = 0$ and $z = 2$. (16)

13. (a) (i) If $f(z) = u + iv$ is a regular function of z, then show that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^p = p^2 |f(z)|^{p-2} |f'(z)|^2 .$$ (8)

(ii) Find the bilinear transformation which maps x = 1, i , -1 respectively onto w = i, 0, -i. . (8)

Or

(b) (i) Find the image of $|z - 3i| = 3$ under the mapping $w = \frac{1}{z}$. (8)

(ii) Find the bilinear transformation which maps the points $z = 0, -i, -1$ into $w = i, 1, 0$. (8)

14. (a) (i) Evaluate $\int_c \frac{z\,dz}{(z-1)(z-2)^2}$ where C is $|z-2| = \frac{1}{2}$ by using cauchy's Integral formula. (8)

(ii) Evaluate $\int_0^\infty \frac{x^2\,dx}{(x^2+a^2)(x^2+b^2)}$, $a > 0, b > 0$ using contour integration. (8)

Or

(b) (i) Find Laurent's series expansion of $f(z) = \frac{7z-2}{z(z-2)(z+1)}$ valid $1 < |z+1| < 3$ (8)

. (ii) Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$. (8)

15. (a) (i) Find the Laplace transform of a periodic function

$f(t) = \begin{cases} t & 0 < t < 1 \\ 2-t & 1 < t < 2 \end{cases}$ and $f(t) = f(t+2)$. (8)

(ii) Using Laplace transform, solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3t}$, given $y = 2$ and $\frac{dy}{dx} = 3$ when $t = 0$. (8)

Or

(b) (i) Use convolution theorem to find inverse Laplace transform of $\frac{s}{(s^2+a^2)^2}$ (8)

(ii) Solve using Laplace transform $\frac{d^2y}{dx^2} + 9y = 18t$ given that $y(0) = 0$ and $y\left(\frac{\pi}{2}\right) = 0$. (8)