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Question Paper Code: U2M02

B.E./B.Tech. DEGREE EXAMINATION, MAY 2022

Second Semester

Mechanical Engineering

21UMA202 - Calculus, Fourier Series And Numerical Methods

(Regulations 2021)

Duration: Three hours

Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

- Iteration method converges if $|g'(x)|$ _____ CO6-U
(a) >1 (b) <1 (c) $=0$ (d) >0
- Newton's method also called _____ method. CO6-U
(a) tangents (b) slope (c) secants (d) false
- The complementary function of $(4D^2 - 3D - 1)y = 2 \sin 2x$ is _____. CO2-App
(a) $Ae^x + Be^{-\frac{x}{4}}$ (b) $Ae^{-x} + Be^{5x}$ (c) $(A+Bx)e^{2x}$ (d) $Ae^x + Be^{4x}$
- $\frac{1}{D^2}(\cos x) =$ _____ CO2-App
(a) $\sin x$ (b) $-\cos x$ (c) $\cos x$ (d) $\tan x$
- Divergence of vector $x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ at $(1, 2, -3)$ is _____. CO3- App
(a) 4 (b) -4 (c) -3 (d) 0
- If $\phi = x^2 + y^2 - z - 10$ then $|\nabla \phi|$ at $(1, 1, 1)$ is _____. CO3- App
(a) $2\vec{i} + 2\vec{j} + \vec{k}$ (b) $2\vec{i} + 2\vec{j} - \vec{k}$ (c) 3 (d) 9
- $\cos x$ is a periodic function with period ----- CO4-App
(a) π (b) 2π (c) $\pi/3$ (d) $2\pi/3$

8. If $f(-x) = -f(x)$, then $f(x)$ is said to be an _____. CO6-U
 (a) Odd Function (b) Even Function (c) Periodic function (d) Self Reciprocal
9. Convolution theorem on Fourier Transform is $F[f(x)*g(x)] =$ _____ CO6-U
 (a) $F(s).G(s)$ (b) $f(s).g(s)$ (c) $F(s)*G(s)$ (d) $f(s)*g(s)$
10. If $f(x)$ is an even function then $\int_{-a}^a f(x)dx =$ _____ CO6-U
 (a) 0 (b) $\frac{1}{2} \int_0^a f(x)dx$ (c) $2 \int_0^a f(x)dx$ (d) $\int_0^a f(x)dx$

PART – B (5 x 2= 10Marks)

11. State Newton's Iterative formula. CO6-U
12. Find the complete solution of $(D^3 - 6D^2 + 11D - 6)y = 0$ CO2-App
13. Show that $\nabla (r^n) = nr^{n-2}\vec{r}$. CO3-App
14. Find b_n in the Fourier series of $f(x) = |\cos x|$ in $(0, 2\pi)$. CO4-App
15. Find the Fourier Sine Transform of $f(x) = e^{-x}, x > 0$ CO5-App

PART – C (5 x 16= 80Marks)

16. (a) (i) Solve for a positive root of $3x - \cos x - 1 = 0$ by Newton's Raphson method. CO1-App (8)
 (ii) Solve $4x + 2y + z = 14, x + 5y - z = 10, x + y + 8z = 20$ by Gauss Elimination method CO1- App (8)
- Or
- (b) (i) Using Power method find numerically largest Eigen value of CO1- App (8)

$$\begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$$

 (ii) Solve $28x + 4y - z = 32; x + 3y + 10z = 24; 2x + 17y + 4z = 35$ by Gauss Seidal method. CO1- App (8)
17. (a) (i) Using method of variation of parameters solve CO2-App (8)
 $(D^2 + a^2)y = \sec ax$
 (ii) Solve $(D^2 - D - 6)y = 3e^{4x}$ CO2-App (8)

Or

- (b) (i) Solve $(D^2 - 3D + 2)y = 2e^x + 2\cos 2x$ CO2 -App (8)
- (ii) If the population of a country double in 50 years, in how many years will it triple under the assumption that the rate of increase of proportional to the number of inhabitants? CO2 -App (8)
18. (a) Verify Gauss divergence theorem for the vector function $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ over the cube bounded by $x = 0, y = 0, z = 0$ and $x = 1, y = 1, z = 1$ CO3-App (16)
- Or
- (b) (i) Prove that $\vec{F} = (x^2 + xy^2)\vec{i} + (y^2 + x^2y)\vec{j}$ is irrotational vector and find the Scalar potential such that $\vec{F} = \nabla\phi$. CO3-App (8)
- (ii) Evaluate Green's theorem for $\int (x^2 - y^2)dx + 2xydy$, where C is bounded by $x = 0, x = a, y = 0$ and $y = b$ CO3-App (8)
19. (a) Find the Fourier series of $f(x) = x + x^2$ in $(-\pi, \pi)$ of periodicity 2π . CO4-App (16)
- Or
- (b) (i) The table of values of the function $y = f(x)$ is given below: CO4-App (8)
- | | | | | | | |
|----|---|---|----|---|---|---|
| x: | 0 | 1 | 2 | 3 | 4 | 5 |
| Y: | 4 | 8 | 15 | 7 | 6 | 2 |
- Find a Fourier series up to the third harmonic to represent $f(x)$ in terms of x .
- (ii) Find the half range sine series for $f(x) = x^2$ in $(0, l)$ CO4-App (8)
20. (a) Find the Fourier transform of $f(x) = \begin{cases} 1 - |x|, & \text{if } |x| \leq 1 \\ 0 & \text{if } |x| > 1 \end{cases}$ and hence deduce that $\int_0^{\infty} \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$. CO5-App (16)
- Or
- (b) (i) Find the Fourier sine & cosine transform of e^{-ax} CO5- App (8)
- (ii) Evaluate $\int_0^{\infty} \frac{dx}{(x^2 + 25)^2}$ CO5- App (8)

