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**Reg. No. :** | | | | | | | | | | |

# Question Paper Code: 54024

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2022
Fourth Semester
Electronics and Communication Engineering
15UMA424 - PROBABILITY AND RANDOM PROCESSES
(Common to Biomedical Engineering)
(Regulation 2015)
(Statistical tables may be permitted)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The moment generating function of the Binomial distribution is CO1- R

(a) $e^{\lambda\left(e^{t}-1\right)}$ (b) $\left(p+q\right)^{n}$ (c) $\left(pe^{t}+q\right)^{n}$ (d) $\left(qe^{t}+p\right)^{n}$

2. If the cumulative distribution function of a random variable X is $F\left(x\right)=1-\left(1+x\right)e^{-x},\ x>0$, then the probability density function of a random variable is CO1- R

(a) $x\,e^{-x}$ (b) $1+x\,e^{-x}$ (c) $x+e^{-x}$ (d) $e^{-x}$

3. The correlation coefficient of two independent random variables X and Y is CO2- R

(a) 1 (b) 0 (c) E(X)E(Y) (d) E(XY)

4. The angle between the two regression lines are CO2- R

(a) $\tan\theta=\frac{1-r^{2}}{r}\left(\frac{\sigma_X\sigma_Y}{\sigma_X^{2}+\sigma_Y^{2}}\right)$ (b) $\tan\theta=\left(\frac{\sigma_X\sigma_Y}{\sigma_X^{2}+\sigma_Y^{2}}\right)$

(c) $\tan\theta=\frac{r}{1-r^{2}}\left(\frac{\sigma_X\sigma_Y}{\sigma_X^{2}+\sigma_Y^{2}}\right)$ (d) $\tan\theta=0$

5. A random process is wide sense stationary process, if CO3- R

(a) E(X(t)) = constant & $R_{XX}(\tau)=$ interms of $\tau$ (b) E(X(t)) = constant only

(c) $R_{XX}(\tau)=$ interms of $\tau$ only (d) $R_{XX}(\tau)=$ constant

6. A random process X(t) is the input to a linear system whose impulse function is $h(t) = 2e^{-t}; t \geq 0$. The autocorrelation of the function of the process is $R_{XX}(\tau) = e^{-2|\tau|}$.Find the power spectral density of the output process Y(t). CO3- R

(a) 1/2 (b) 1/3 (c) 2/3 (d) 3/2

7. If the auto correlation function for a stationary ergodic process with no periodic components is $R(\tau) = 25 + \frac{4}{1+6\tau^2}$, the mean square value of the process is CO4- R

(a) 29 (b) 25 (c) 5 (d) 4

8. If the power spectral density function is $S(\omega)$, then the auto correlation function is CO4- R

(a) $\frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega)e^{i\omega\tau}\,d\omega$ (b) $\frac{1}{\pi}\int_{0}^{\infty} S(\omega)e^{i\omega\tau}\,d\omega$ (c) $\frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega)e^{-i\omega\tau}\,d\omega$ (d) $\int_{-\infty}^{\infty} S(\omega)e^{i\omega\tau}\,d\omega$

9. Let X(t) be the input voltage to a circuit system & Y(t) be the output voltage. If X(t) is a stationary random process with $\mu_X = 0$ then $\mu_Y$ is CO5- R

(a) 1 (b) 0 (c) 5 (d) -1

10. The power transfer function of the system is CO5- R

(a) $H(\omega)$ (b) $h(t)$ (c) $|H(\omega)|$ (d) $|H(\omega)|^2$

PART – B (5 x 2= 10Marks)

11. If $f(x) = \begin{cases} ke^{-x}, x > 0 \\ 0, otherwise \end{cases}$, is the pdf of a random variable X, then find the value of k. CO1-App

12. If the joint probability density function of (X, Y) is CO2 -App

$f(x,y) = \begin{cases} \frac{1}{4}, \ 0 < x, \ y < 2 \\ 0, \ otherwise \end{cases}$, then find $P(X + Y \leq 1)$

13. What is Markov Process? CO3- R

14. Prove that $R_{XY}(\tau) = R_{YX}(-\tau)$ CO4- R

15. Check whether the system $y(t) = x^3(t)$ is linear or not. CO5- R

PART – C (5 x 16= 80 Marks)

16. (a) (i) A continuous random variable X has the density function CO1- App (8)

$f(x) = \frac{k}{1+x^2}, -\infty < x < \infty$, find the value of k and the cumulative distribution function.

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| | | (ii) State and Prove the memory less property of Geometric distribution. | CO1- App | (8) |
| | | Or | | |
| | (b) | (i) Find the moment generating function of uniform distribution and hence find its mean and variance. | CO1- App | (8) |
| | | (ii) In a normal distribution 31% of items are under 45 and 8% of items are over 64. Find the mean and Standard deviation of the distribution. | CO1- App | (8) |
| 17. | (a) | (i) The joint probability mass function of (X,Y) is given by $P(x,y)=\frac{1}{72}(2x+3y),\ x=0,1,2$ and $y=1,2,3$. Find all the marginal and conditional probability functions of X & Y. | CO2- App | (10) |
| | | (ii) The joint probability density function of (X,Y) is $f(x,y)=e^{-(x+y)},\ x,y\geq 0$. Are X and Y are Independent? | CO2- App | (6) |
| | | Or | | |
| | (b) | (i) The joint probability density function of a random variable(X,Y) is $f(x,y)=25\ e^{-5y},\ 0<x<0.2,\ y>0$. Find the covariance of X &Y. | CO2- App | (8) |
| | | (ii) The random variables X and Y each follow exponential distribution with parameters1 and are independent. Find the probability density function of U = X-Y. | CO2- App | (8) |
| 18. | (a) | (i) If the random process $\{X(t)\}$ takes the value -1 with probability 1/3 and takes the value 1 with probability 2/3, find whether $\{X(t)\}$ is a stationary process or not. | CO3- Ana | (8) |
| | | (ii) A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again, as he is to travel by train. Now suppose that on the first day of the week, the man tossed the fair die and drove to work if and only if six appeared. Find<br>(a) the probability that he takes a train on the third day and<br>(b) the probability that he drives to work in the long run | CO3- Ana | (8) |
| | | Or | | |
| | (b) | (i) Prove that the random telegraph process $\{Y(t)\}$ is a wide sense stationary process. | CO3- Ana | (8) |

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| | | (ii) Three boys A, B, C are throwing ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as equally likely to throw the ball to B or to A. Show that the process is Markovian. Find the transition matrix and classify the states. | CO3- Ana | (8) |
| 19. | (a) | (i) Consider two random processes $X(t) = 3\cos(\omega t + \theta)$ and $Y(t) = 2\cos\left(\omega t + \theta - \frac{\pi}{2}\right)$ where $\theta$ is a random variable uniformly distributed in $(0, 2\pi)$. Prove that $\sqrt{R_{XX}(0)R_{YY}(0)} \geq \lvert R_{XY}(\tau)\rvert$ | CO4- App | (10) |
| | | (ii) The cross power spectrum of real random process $\{X(t)\}$ and $\{Y(t)\}$ is given by $S_{XY}(\omega) = \begin{cases} a + jb\omega, & \lvert\omega\rvert < 1 \\ 0, & elsewhere \end{cases}$. Find the cross correlation function. | CO4- App | (6) |
| | | Or | | |
| | (b) | (i) Find the mean square value of the processes whose power spectral density is given by $S_X(\omega) = \frac{1}{\omega^4 + 10\omega^2 + 9}$ | CO4- App | (8) |
| | | (ii) Find the auto correlation of the process $\{X(t)\}$ whose spectral density is given by $S_X(\omega) = \begin{cases} 1 + \omega^2, & \lvert\omega\rvert \leq 1 \\ 0, & \lvert\omega\rvert > 1 \end{cases}$ | CO4- App | (8) |
| 20. | (a) | A Linear system is described by the impulse response $h(t) = \beta e^{-\beta t} u(t)$. Assume an input process whose auto correlation function is $B\delta(\tau)$. Find the mean and Auto Correlation function of the output process. | CO5- App | (16) |
| | | Or | | |
| | (b) | A random process $\{X(t)\}$ is the input to a linear system whose impulse response is given by $h(t) = 2e^{-t}, t \geq 0$. If the auto correlation function of the process $X(t)$ is $R_{XX}(\tau) = e^{-2\lvert\tau\rvert}$, determine the cross correlation function $R_{XY}(\tau)$ and $R_{YX}(\tau)$ between the input process $X(t)$ and the output process $Y(t)$ | CO5- App | (16) |