A   Reg. No. :

Duration: Three hours   Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x - 1} =$ CO1-R

    (a) 0   (b) 1   (c) 2   (d) 3

2. Suppose $f(x) = \begin{cases} \dfrac{x^2 - x}{2x} & if \ x \neq 0 \\ k & if \ x = 0 \end{cases}$. If $f(x)$ is continuous at x=0, then   CO1-R

    the value of 'k' is

    (a) -1   (b) 1   (c) $-\dfrac{1}{2}$ (d) $\dfrac{1}{2}$

3. If $u = (x - y)(y - z)(z - x)$ then $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} =$ CO2-R

    (a) 1   (b) 0   (c) 3   (d) 6

4. If $x = r \cos \theta, y = r \sin \theta$ find $\dfrac{\partial(x, y)}{\partial(r, \theta)}$ CO2-R

    (a) r   (b) r^2 (c) 0   (d) 2

5. $\Gamma\left(\dfrac{1}{2}\right) =$ CO3-R

    (a) 1   (b) $\dfrac{1}{2}$ (c) $\sqrt{\pi}$ (d) π

6. $\displaystyle\int_0^{\frac{\pi}{2}} \cos^8 x \, dx =$ CO3-R

(a) $\dfrac{35\pi}{256}$ (b) $\dfrac{256}{35\pi}$ (c) 35π (d) 256

7. Value of the double integral $\int_0^1 \int_0^y dy\,dx$ is   CO4-R

(a) 0   (b) $\dfrac{1}{2}$ (c) $\dfrac{3}{2}$ (d) $\dfrac{3}{4}$

8. $\displaystyle\int_0^{\frac{\pi}{2}} \int_0^{\sin\theta} r\,d\theta\,dr =$ CO4-R

(a) $\dfrac{1}{8}$ (b) $\dfrac{\pi}{2}$ (c) $\dfrac{\pi}{8}$ (d) π

9. The product of two eigen values of $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ is 16 then the   CO5-R

third eigen value is

(a) 3   (b) 2   (c) 4   (d) 5

10. If the product of two eigenvalues of third order singular matrix A is 34, then the third eigenvalue of the matrix A is   CO5-R

(a) 3   (b) -1   (c) 1   (d) 0

PART – B (5 x 2= 10Marks)

11. Find   CO1-R

$\dfrac{dy}{dx}$ given $x = a\cos^3\theta;\; y = a\sin^3\theta.$

12. State Euler's theorem on homogeneous functions.   CO2-U

13. Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \sin^4 x \cos^2 x \, dx$ CO3-R

14. Prove that $\displaystyle\int_{-1}^{3} \int_{x^2}^{2+x} dy\,dx = \dfrac{8}{3}.$ CO4-R

15. State Cayley Hamilton theorem.   CO5-U

2

51002

PART – C (5 x 16= 80Marks)

16. (a) (i) Find $\frac{dy}{dx}$ if $y = \tan^{-1} \sqrt{\frac{1 - \sin x}{1 + \sin x}}$ CO1-App   (16)

Or

(b) (i) If $y = e^{ax} \sin bx$, prove that $\frac{d^2 y}{dx^2} - 2a \frac{dy}{dx} + \left(a^2 + b^2\right) y = 0$ CO1-App   (8)

(ii) Obtain the Maclaurin's series for $\log(1 + x)$ CO1-App   (8)

17. (a) (i) Verify Euler's theorem for the function $u = \sin^{-1} \frac{x}{y} + \tan^{-1} \frac{y}{x}$ CO2-App   (8)

(ii) Given the transformations $u = e^x \cos y \; and \; v = e^x \sin y$ and that   CO2-App   (8)
ϕ is a function of u and v and also of x and y, prove that

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = (u^2 + v^2) \left[\frac{\partial^2 \phi}{\partial u^2} + \frac{\partial^2 \phi}{\partial v^2} \right]$$

Or

(b) Given the transformation $u = e^x \cos y \; and \; v = e^x \sin y$ and that   CO2-App   (16)
f is a function of u and v and also of x and y, prove that
$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = (u^2 + v^2) \left(\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2} \right)$$

18. (a) (i) Evaluate $\int \frac{1}{1 + \cos x} dx$ CO3-App   (8)

(ii) Evaluate $\int e^{2 \log x} e^{x^3} dx$ CO3-App   (8)

Or

(b) Prove that   CO3-App   (16)

$$\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

51002

19. (a) Change the order of integration $\int_0^1 \int_{y^2}^{2-y} xy\,dy\,dx$ and hence evaluate it   CO4-App   (16)

<div align="center">Or</div>

(b) (i) Find the smaller area bounded by the parabolas $y^2 = 4 - x$ and $y^2 = 4 - 4x$.   CO4-App   (8)

(ii) Find the volume of the Sphere $x^2 + y^2 + z^2 = a^2$.   CO4-App   (8)

20. (a) (i) Find the eigen values and eigen vectors of the matrix   CO5-App   (8)

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

(ii) Using Cayley Hamilton's theorem find the inverse of the matrix   CO4-App   (8)

$$\begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

<div align="center">Or</div>

(b) Reduce the Q.F $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4zx$ in to a canonical form by an orthogonal transformation.   CO5-App   (16)

51002