**Reg. No. :** | | | | | | | | | | |

# Question Paper Code: 41002

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2022

First Semester

Civil Engineering

14UMA102 - ENGINEERING MATHEMATICS – I

(Common to ALL branches)

(Regulation 2014)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions.

PART A - (10 x 1 = 10 Marks)

1. If 1 and 2 are the eigen values of 2x2 matrix A. what are the eigen values of $A^2$.

   (a) 1 & 2 (b) 1 & 4 (c) 2 & 4 (d) 2 & 3

2. $\begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} =$

   (a) 0 (b) 1 (c) 2 (d) 3

3. Examine the nature of the series $1 + 2 + 3 + 4 + \ldots\ldots\ldots + n + \ldots\ldots\infty$

   (a) divergent (b) convergent (c) oscillatory (d) linear

4. The geometric series $1 + r + r^2 + r^3 + \ldots\ldots + r^n + \ldots\ldots$ converges if

   (a) $r \leq 1$ (b) $r \geq 1$ (c ) $r > 1$ (d ) $r < 1$

5. What is the radius of curvature at (3, 4) on the curve $x^2 + y^2 = 25$?

   (a) 5 (b) -5 (c) 25 (d) -25

6. The envelope of the family of straight lines $y = mx + \frac{1}{m}$, m being the parameter is

(a) $y^2 = -4x$ (b) $x^2 = 4y$ (c) $y^2 = 4x$ (d) $x^2 = -4y$

7. Let $u$ and $v$ be functions of $x$, $y$ and $u = e^v$ .Then $u$ and $v$ are

(a) Functionally dependent (b) Functionally independent
(c) Functionally linear (d) Functionally non-linear

8. A stationary point of $f(x, y)$ at which $f(x, y)$ has neither a maximum nor a minimum is called

(a) Extreme point (b) Max-Min point
(c) Saddle point (d) Nothing can be said

9. $\int_0^1\int_0^2\int_0^3 xyz\,dx\,dy\,dz$

(a) 9 (b) $\frac{9}{4}$ (c) $\frac{9}{2}$ (d) $\frac{1}{9}$

10. By changing the order of integration, we get $\int_0^1\int_0^y f(x, y)\,dx\,dy$ =

(a) $\int_0^1\int_0^x f(x, y)\,dy\,dx$ (b) $\int_0^1\int_x^1 f(x, y)\,dy\,dx$ (c) $\int_0^1\int_y^1 f(x, y)\,dx\,dy$ (d) $\int_0^1\int_x^0 f(x, y)\,dy\,dx$

PART - B (5 x 2 = 10 Marks)

11. Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$.

12. Test the convergence of the series $\sum_{1}^{\infty} \frac{n!2^n}{n^n}$ by D'Alembert's Ratio test.

13. Find the radius of curvature of the curve $y = e^x$ at $x = 0$.

14. If $x = u^2 - v^2$ and $y = 2uv$ , find the Jacobian of $x$ and $y$ with respect to $u$ and $v$.

15. Evaluate $\int_0^2 \int_0^{\pi} r \sin^2\theta \, d\theta \, dr$ .

PART - C (5 x 16 = 80 Marks)

16. (a) Diagonalize the matrix by orthogonal transformation $\begin{bmatrix} 10 & -2 & -5 \\ -2 & 2 & 3 \\ -5 & 3 & 5 \end{bmatrix}$. (16)

Or

(b) Reduce the Q.F $x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$ in to a canonical form by an orthogonal transformation. (16)

17. (a) (i) Show that the sum of the series $\frac{15}{16} + \frac{15}{16} \times \frac{21}{24} + \frac{15}{16} \times \frac{21}{24} \times \frac{27}{32} + \cdots \infty = \frac{47}{9}$. (8)

(ii) Show that the series $1 - 2 + 3 - 4 + \dots \infty$ oscillates infinitely. (8)

Or

(b) Prove that if $b - 1 > a > 0$, the series $1 + \frac{a}{b} + \frac{a(a+1)}{b(b+1)} + \frac{a(a+1)(a+2)}{b(b+1)(b+2)} + \dots\dots$ converges. (16)

18. (a) (i) Find the radius of curvature at any point of the cycloid $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$. (8)

(ii) Find the evolute of the parabola $y^2 = 4ax$. (8)

Or

(b) (i) Find the evolute of the curve $x = a(\cos t + t \sin t)$, $y = a(\sin t - t\cos t)$. (8)

(ii) Find the envelope of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a and b are connected by the relation $a^2 + b^2 = c^2$, $c$ being a constant. (8)

19. (a) (i) Find the Taylor's series of $e^x \log(1 + y)$ in powers of $x$ and $y$ up to third degree terms. (8)

(ii) Find the maximum value of $f(x, y) = \sin x \sin y \sin(x + y)$; $0 \le x, y < \pi$. (8)

Or

(b) (i) If $g(x, y) = \psi(u, v)$, where $u = x^2 - y^2$, $v = 2xy$, then prove that

$$\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} = 4(x^2 + y^2)\left(\frac{\partial^2 \psi}{\partial u^2} + \frac{\partial^2 \psi}{\partial v^2}\right).$$ (8)

(ii) Find the Jacobian of $y_1, y_2, y_3$ with respect to $x_1, x_2, x_3$ where $y_1 = \frac{x_2 x_3}{x_1}$, $y_2 = \frac{x_1 x_3}{x_2}$, $y_3 = \frac{x_1 x_2}{x_3}$. (8)

20. (a) (i) Change the order of integration and evaluate $\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$. (8)

(ii) Evaluate $\iiint \frac{dx\ dy\ dz}{\sqrt{1 - x^2 - y^2 - z^2}}$ for all positive values of *x, y, z* for which the integral is real. (8)

Or

(b) (i) Evaluate $\iint_S z^3 dS$, where is S is the positive octant of the surface of the sphere. (8)

(ii) Evaluate $\iiint_V xyz\,dx\,dy\,dz$, where V is the volume of space inside the tetrahedron bounded by the planes x = 0, y = 0, z = 0 and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. (8)

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