A **Reg. No. :** | | | | | | | | | | | | |

Question Paper Code: U2M06

B.E./B.Tech. DEGREE EXAMINATION, MAY 2022

Second Semester

Civil Engineering

21UMA206- Differential Equations, Complex Analysis & Transform Techniques

(Regulations 2021)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. $\frac{1}{(D-m)^2}e^{mx}$ = _____ CO1-App

   (a) xe^{mx} (b) x^2e^{mx} (c) $\frac{x^2}{2}e^{mx}$ (d) $\frac{x^2}{m}e^{mx}$

2. The complementary function of $(4D^2 - 3D - 1)y = 2\sin 2x$ is _____ CO1-App

   (a) $Ae^x + Be^{-\frac{x}{4}}$ (b) $Ae^{-x}+Be^{5x}$ (c) $(A+Bx)e^{2x}$ (d) $Ae^{x}+Be^{4x}$

3. If $\varphi = x^2 + y^2 - z - 10$ then $|\nabla \varphi|$ at (1, 1, 1) is _____ CO2-App

   (a) $2(\vec{i} + \vec{j} + \vec{k})$ (b) $2\vec{i} + 2\vec{j} - \vec{k}$ (c) 3 (d) 9

4. If φ is a vector point function then Curl (gradφ) = CO2-App

   (a) 1 (b)0 (c)2 (d) None of the above

5. The PDE obtained from z = (x+a)(y+b) is __. CO3- App

   (a) 3z = px + qy (b) py - qx = 0 (c) z = pq (d) px+qy = 0

6. The PDE of all planes having equal intercepts on the X axis and Y axis is _____ CO3- App

   (a) p = q (b) p + q = 0 (c) pq = 1 (d) p(q + 1) = q

7. Simple pole is a pole of order _____ CO6-U

   (a) 1 (a) 2 (a) 3 (a) 4

8. The poles of z cot z is _____ CO6-U

(a) 0 (b) $\pm n\pi$ (c) 1 (d) π

9. L (sin h at) = _____ CO6-R

(a) $\frac{s}{s^2 - a^2}$ (b) $\frac{a}{s^2 - a^2}$ (c) $\frac{s}{s^2 + a^2}$ (d) $\frac{a}{s^2 + a^2}$

10. L [t f(t)] = _____ CO6-R

(a) F'(s) (b)–F'(s) (c)F(s) (d) –F(s)

PART – B (5 x 2= 10Marks)

11. Compute the particular integral for $(D^2 - 2D + 1)y = \cosh x$. CO1-App

12. Compute the unit normal vector to the surface $x^2 + y^2 + z^2 = 1$ at (1, 1, 1). CO2-App

13. Compute the complete integral of $p - q = k$ CO3-App

14. State Cauchy's residue theorem. CO6-R

15. Verify initial value theorem for the function $1+e^{-2t}$. CO6-U

PART – C (5 x 16= 80Marks)

16. (a) (i) Using method of variation of parameters solve $(D^2 + 4)y = \sec 2x$. CO1-App (8)

(ii) A colony of bacteria is growing exponentially. At time t=0 it has 10 bacteria in it, and at time t=4 it has 2000. At what time will it have 100,000 bacteria? CO1- App (8)

Or

(b) (i) Solve: $(x^2D^2 + xD + 1)y = x \sin(\log x)$ CO1- App (8)

(ii) Solve: $(D^2 - 4D + 3)y = \sin 3x + e^{2x}$ CO1- App (8)

17. (a) Verify Divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - xz)\vec{j} + (z^2 - xy)\vec{k}$ over the rectangular parallelepiped x = 0, x = a, y = 0, y = b, z = 0, z = c. CO2-App (16)

Or

(b) (i) Using Green's theorem, Evaluate $\int_C (3x^2 - 8y^2)\,dx + (4y - 6xy)dy$ where C is the boundary of the region defined by X = 0, Y = 0, X + Y= 1 in the XY plane. CO2 -App (8)

(ii) Prove that $\bar{F} = (x^2 + xy^2)\vec{\imath} + (y^2 + x^2y)\vec{\jmath}$ is irrotational vector and compute the Scalar potential such that $\bar{F} = \nabla\emptyset$. CO2 -App (8)

18. (a) (i) Solve: $(mz - ny)\,p + (nx - lz)\,q = ly - mx$ CO3-App (8)

(ii) Solve $(D^2 - DD' - 3D'^2)z = \sin(x + y) + e^{6x+y}$ CO3-App (8)

Or

(b) (i) Solve $p^2 + q^2 = x^2 + y^2$ CO3-App (8)

(ii) Form a PDE by eliminating arbitrary functions from CO3-App (8)

$z = px + qy + p^2 - q^2$

19. (a) CO4-App (8)

(i) Evaluate $f(z) = \int_C \frac{\cos \pi z^2 + \sin \pi z^2}{(z-1)(z-2)} dz$ by using Cauchy's

Integral formula where C is $|z| = 3$

(ii) Expand $\frac{z-1}{(z+2)(z+3)}$ as Laurent's series valid in the region CO4-App (8)

$2 < |z| < 3$

Or

(b) Using Contour integration, to prove CO4-App (16)

$$\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + a^2)(x^2 + b^2)} dx = \frac{\pi}{a+b} \quad a > b > 0$$

20. (a) (i) Solve the differential equation $\frac{d^2y}{dt^2} + y = \sin 2t \; ; y(0) = 0 \; ; y'(0) = 0$ by using Laplace transform method. CO5-App (8)

(ii) Compute the Laplace Transforms of $\frac{\cos at - \cos bt}{t}$ CO5-App (8)

Or

(b) (i) Using the periodic function, Compute the Laplace transform of CO5-App (8)

f(t) = $f(t) = \begin{cases} k, & 0 \le t \le a \\ -k, & a \le t \le 2a \end{cases}$

(ii) Using Convolution Theorem, Compute $L^{-1}\left[\frac{1}{(s^2 + a^2)(s^2 + b^2)}\right]$ CO5-App (8)