Reg. No. : | | | | | | | | | | |

Question Paper Code: 43021

B.E. / B.Tech. DEGREE EXAMINATION, AUGUST 2021

Third Semester

Civil Engineering

14UMA321 - TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to ALL branches)

(Regulation 2014)

Duration: 1:45 hour Maximum: 50 Marks

PART A - (10 x 2 = 20 Marks)

(Answer any ten of the following questions)

1. Find the Fourier constants b_n for $x \sin x$ in $(-\pi, \pi)$.

2. Find the Fourier sine transform of $\frac{1}{x}$.

3. Find the Z transform of $x(n) = \begin{cases} \frac{a^n}{n!} & \text{for } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$

4. Classify $4u_{xx} + 4u_{xy} + u_{yy} - 6u_x - 8u_y - 16u = 0$.

5. Derive the explicit difference equation corresponding to the partial differential equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$.

6. If $f(x) = |x|$, expanded as a Fourier series in $(-\pi, \pi)$, find a_0.

7. If $F(f(x)) = F(s)$, then prove that $F\left[\frac{d^n(f(x))}{dx^n}\right] = (-is)^n F(s)$

8. State initial and final value theorems of Z transforms.

9. Evaluate the steady state temperature of a rod of length ℓ whose ends are kept at 30^o and $40^o c$.

10. Derive the explicit difference equation corresponding to the partial differential equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$.

11. Find the Fourier constants b_n for $x \sin x$ in $(-\pi, \pi)$.

12. Find the Fourier sine transform of $\frac{1}{x}$.

13. State initial and final value theorem of Z – transform.

14. Define steady state condition on heat flow.

15. Write down the standard five-point formula to solve the Laplace equation

PART – B (3 x 10= 30 Marks)

(Answer any three of the following questions)

11. Find the Fourier series for $f(x) = 1 + x + x^2$ in $(-\pi, \pi)$. Deduce that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\infty = \frac{\pi^2}{6}$ (10)

12. Find Fourier transform of $e^{-a^2x^2}$, $a>0$ and hence show that $e^{-x^2/2}$ is self-reciprocal. (10)

13. Find $Z\,(t^2 e^{-t})$ and $Z\,(\sin^3 \frac{n\pi}{6})$ (10)

14. A string is stretched between two fixed points at a distance 2ℓ apart and the points of the string are given initial velocities v where,

$$V = \begin{cases} \frac{cx}{l} \text{ in } 0 < x < l \\ \frac{c(2l - x)}{l} \text{ in } l < x < 2l \end{cases}$$

, x being the distance from an end point. Find the displacement of the string at any subsequent time. (10)

15. Solve $U_{xx} + U_{yy} = 0$, over the square mesh of side 4 units satisfying the following boundary conditions, by using Liebmann's iteration method by taking $h = k = 1$
    - (i) $U(0,y) = y^2/4$ for $0 \le y \le 4$
    - (ii) $U(4,y) = y^2$ for $0 \le y \le 4$
    - (iii) $U(x,0) = 0$ for $0 \le x \le 4$
    - (iv) $U(x,4) = 8 + 2x$ for $0 \le x \le 4$ (10)