**Reg. No. :** | | | | | | | | | | |

## Question Paper Code: 33021

B.E. / B.Tech. DEGREE EXAMINATION, AUGUST 2021

Third Semester

Civil Engineering

01UMA321 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to ALL Branches)

(Regulation 2013)

Duration: 1:45 hour Maximum: 50 Marks

PART A - (10 x 2 = 20 Marks)

**(Answer any ten of the following questions)**

1. Find the constant term in the Fourier series corresponding to $f(x) = \sqrt{1 - \cos x}$ expressed in the interval $(-\pi, \pi)$.
2. State the conditions for $f(x)$ to have Fourier series expansion.
3. Find the Fourier cosine transform of $e^{-2x}$.
4. Find the Fourier transform of $f(x) = \begin{cases} 1 & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$.
5. Find the Z-transform of $a^n$.
6. Write the formula for $Z^{-1}[F(z)]$ using Cauchy's residue theorem.
7. State initial and final value theorems on $z$ - transform.
8. What does $a^2$ represent in the equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$ ?
9. Write down the diagonal five point formula in Laplace equation.
10. State the diagonal five point formula to solve the equation $u_{xx} + u_{yy} = 0$.
11. State Parseval's theorem in Fourier series.

12. If the Fourier series of the function $f(x) = x + x^2$ in the interval $-\pi \le x \le \pi$ is $\frac{\pi^2}{3} + \sum_{n=1}^{\infty} (-1)^n \left[ \frac{4}{n^2} \cos nx - \frac{2}{n} \sin nx \right]$, then find the value of the infinite series $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$

13. Prove that if $F(s)$ is the Fourier transform of $f(x)$, then $F\{ f(x-a) \} = e^{isa} F(s)$.

14. Find the Fourier transform of $f(x) = \begin{cases} 1 & |x| \le 1 \\ 0 & |x| > 1 \end{cases}$

15. Find $Z$ transform of $a^n$.

PART – B (3 x 10= 30 Marks)

**(Answer any three of the following questions)**

16. Expand the function $f(x) = \sin x, 0 < x < \pi$ in a Fourier cosine series. (10)

17. Find the Fourier cosine transform of $e^{-4x}$ and hence find the values of $\int_0^\infty \frac{\cos 2x}{x^2 + 16} dx$ and $\int_0^\infty \frac{x \sin 2x}{x^2 + 16} dx$ (10)

18 Using Convolution theorem, evaluate $Z^{-1} \left[ \frac{9z^3}{(3z-1)^2 (z-2)} \right]$ (10)

19. A tightly stretched string of length $l$ with fixed ends is initially in its equilibrium position. It is set vibrating by giving each point a velocity $v_0 \sin^3 \left( \pi \frac{x}{l} \right)$. Find the displacement $y(x,t)$. (10)

20. Solve $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square mesh with sides $x = 0, \quad y = 0, \quad x = 3, \quad y = 3$ with $u = 0$ on the boundary and mesh length 1 unit. (10)

**33021**