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Question Paper Code: R2M05

B.E./B.Tech. DEGREE EXAMINATION, APRIL / MAY 2025

Second Semester

Electrical and Electronics Engineering

R21UMA205 - CALCULUS AND TRANSFORMS TECHNIQUES

(Regulations R2021)

Duration: Three hours

Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 2 = 20 Marks)

1. Solve the Euler Cauchy's equation $(x^2 D^2 + xD)y = 0$ CO1-App
2. Transform $(x^2 D^2 - 3xD - 5)y = x^2 \sin(\log x)$ into linear equation with constant coefficient CO1-App
3. Show that $\vec{F} = x^2 \vec{i} + y^2 \vec{j} + z^2 \vec{k}$ is a conservative vector field. CO2- App
4. If $\phi = x^2 + y^2 - z - 10$, then find $|\nabla \phi|$ at (1, -1, 1). CO2-App
5. Compute $L\left[\frac{3}{t^2}\right]$ CO3-U
6. State and Prove the Initial value theorem. CO3 -U
7. State Dirichlet's conditions CO4 -U
8. Compute the root mean square value of the function $f(x) = x$ in (0,1) CO4-App
9. Define Fourier cosine transform pair CO5-App
10. State convolution theorem for Fourier transform. CO6 -U

PART – B (5 x 16= 80 Marks)

11. (a) (i) Solve the method of variation of parameters $(D^2 + 4)y = \sec 2x$ CO1- App (8)
(ii) Solve the differential equation $(D^2 - D - 6)y = 3e^{4x} + 5$ CO1-App (8)

Or

- (b) (i) If the population of a country double in 50 years, in how many years will it triple under the assumption that the rate of increase is proportional to the number of inhabitants? CO1- App (8)
- (ii) Solve the differential equation $(x^2 D^2 + 4xD + 2)y = x \log x$ CO1- App (8)
12. (a) Verify Divergence theorem for $\vec{F} = 5x^2 \vec{i} + 4y^2 \vec{j} + 7z^2 \vec{k}$ over the cube $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$. CO2-App (16)
- Or
- (b) Verify Green's theorem in XY plane for, $\int_C (x^2 + y^2) dx - 2xy dy$ CO2-App (16)
- where C is boundary of the region $x = 0, y = 0, x = a, y = a$.
13. (a) (i) Find the Laplace transform of CO3- App (8)
- $$f(t) = \begin{cases} k, & 0 < t < a \\ -k, & a < t < 2a \end{cases}$$
- (ii) Compute $L\left[\frac{e^{-at} - e^{-bt}}{t}\right]$ CO3- App (8)
- Or
- (b) (i) Solve by using L.T. $y'' - 3y' + 2y = e^{-t}$ given that if CO3- App (8)
- $y(0) = 1, y'(0) = 0$
- (ii) Solve by the convolution theorem CO3- App (8)
- $$L^{-1}\left[\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}\right]$$
14. (a) (i) Compute first two harmonics of the Fourier series for the CO4- App (8)
- following data.
- | | | | | | | | |
|---|-----|---------|----------|-------|----------|----------|--------|
| x | 0 | $\pi/3$ | $2\pi/3$ | π | $4\pi/3$ | $5\pi/3$ | 2π |
| y | 1.8 | 0.3 | 0.5 | 2.16 | 1.3 | 1.76 | 1.8 |
- (ii) Find the Half range sine series for $f(x) = x$ in $(0, \pi)$ CO4- App (8)
- Or
- (b) Express $f(x) = (\pi - x)$ as a Fourier series of period 2π in the CO4- App (16)
- interval $0 < x < 2\pi$.

15. (a) Compute the Fourier Transform of CO5-App (16)

$$f(x) = \begin{cases} a - |x| & \text{if } |x| \leq a \\ 0 & \text{if } |x| > a \end{cases} \quad \text{and hence evaluate} \quad \text{(i)} \int_0^{\infty} \left(\frac{\sin x}{x} \right)^4 dx$$

$$\text{(ii)} \int_0^{\infty} \left(\frac{\sin x}{x} \right)^2 dx$$

Or

- (b) Evaluate CO5-App (16)
- (i) $\int_0^{\infty} \frac{dx}{(x^2 + 1)(x^2 + 4)}$ (ii) $\int_0^{\infty} \frac{dx}{(x^2 + 36)^2}$ using Fourier transform

