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Question Paper Code:R2M10

B.E./B.Tech. DEGREE EXAMINATION, APRIL / MAY 2025

Second Semester

R21UMA210- DIFFERENTIAL EQUATIONS, FOURIER SERIES & TRANSFORMS

(Regulations R2021)

(Common to CSD and AI&DS Engineering branches)

Duration: Three hours

Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The auxiliary equation of the equation $x^2 \frac{d^2 y}{dx^2} + 4x \frac{dy}{dx} + 2y = e^x$ is _____. CO 1 – App
(a) $m^2 - 4m + 5 = 0$ (b) $m^2 + 3m - 2 = 0$ (c) $m^2 + 3m + 2 = 0$ (d) $2m^2 + 5m - 7 = 0$
2. The solution of $(D^3 + D^2 - D - 1)y = 0$ is _____. CO 1 – App
(a) $Ae^x + Bxe^x + Cx^2e^x$ (b) $(Ax + B)e^x + Ce^{-x}$
(c) $e^{-x} + (\cos 2x + i \sin 2x)$ (d) $(Ax + B)e^{-x} + Ce^x$
3. $L(\cos at) =$ _____. CO2 – App
(a) $\frac{s}{s^2 - a^2}$ (b) $\frac{a}{s^2 - a^2}$ (c) $\frac{s}{s^2 + a^2}$ (d) $\frac{a}{s^2 + a^2}$
4. $L(e^{iat})$ CO2 – App
(a) $1/s + ia$ (b) $1/s - ia$ (c) $s/s + ia$ (d) $s/s + ia$
5. The Z transform of Unit step sequence is _____. CO 6 – U
(a) $\frac{z}{z + 1}$ (b) 0 (c) 1 (d) $\frac{z}{z - 1}$
6. The inverse Z transform of $\frac{z}{(z - 2)(z - 1)}$ is _____. CO3 – App
(a) $(2^n + 1)u(n)$ (b) $(3^n + 1)u(n)$ (c) $(2^n - 1)u(n)$ (d) $(3^n - 1)u(n)$

7. $\cos x$ is a periodic function with period -----.
- (a) π (b) 2π (c) $\pi/3$ (d) $2\pi/3$ CO 4– App
8. The Fourier constant b_n in $(-\pi, \pi)$ for $x \sin x$ is ____.
- (a) x^2 (b) $3x$ (c) 0 (d) 1 CO 4– App
9. If $F[f(x)] = f(s)$ then the function is said to be _____
- (a) Odd (b) Even (c) Self Reciprocal (d) Periodic CO 6 – U
10. $F_s[e^{-ax}] =$ _____
- (a) $\sqrt{\frac{2}{\pi}} \frac{s}{s^2+a^2}$ (b) $\sqrt{\frac{2}{\pi}} \frac{a}{s^2+a^2}$ (c) $\sqrt{\frac{2}{\pi}} \frac{a^2}{s^2+a^2}$ (d) $\sqrt{\frac{2}{\pi}} \frac{s^2}{s^2+a^2}$ CO 5– App

PART – B (5 x 2= 10 Marks)

11. Solve: $(D - 2)^2 y = e^{2x}$ CO 1 -App
12. Evaluate $L[\cos at]$ CO2– App
13. Evaluate $Z(na^n)$ CO3– App
14. Find a_0 and a_n in the Fourier series of $f(x) = x + x^3$ in $(-\pi, \pi)$. CO4– App
15. Find the Fourier sine transform of $\frac{1}{x}$ CO5– App

PART – C (5 x 16= 80 Marks)

16. (a) (i) Solve $[(x+2)^2 D^2 - (x+2)D + 1]y = x+2$ CO1–App (8)
- (ii) Solve $(x^2 D^2 + 4xD + 2)y = x + \frac{1}{x}$ CO1–App (8)
- Or
- (b) (i) Solve $[(x+1)^2 D^2 + (x+1)D + 1]y = 4 \cos \log(x+1)$ CO1–App (8)
- (ii) Solve $(x^2 D^2 - xD + 1)y = \left(\frac{\log x}{x}\right)^2$ CO1–App (8)
17. (a) (i) Find the Laplace transform of $f(t) = \begin{cases} E, & 0 < t < b \\ -E, & b < t < 2b \end{cases}$ and $f(t+2b) = f(t)$ CO2–App (8)
- (ii) Solve by using convolution theorem $L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$ CO2–App (8)
- Or

- (b) (i) Solve the differential equation $\frac{d^2 y}{dt^2} + 9y = \cos 2t$ if $y(0) = 1$ & $y\left(\frac{\pi}{2}\right) = -1$ by using Laplace transform method. CO2 App (8)

- (ii) Find the Laplace Transforms of $te^{-2t} \sin t$ CO2- App (8)

18. (a) (i) Evaluate $Z[a^n \cos n\theta]$ and $Z[b^n \sin n\theta]$ CO3-App (8)

- (ii) Evaluate $Z^{-1}\left[\frac{z}{z^2 + 4z + 3}\right]$ CO3-App (8)

Or

- (b) (i) Solve the difference equation $y_{n+2} - 6y_{n+1} + 8y_n = 5^n$ given that $y_0 = 0, y_1 = 0$ CO3-App (8)

- (ii) Using Convolution theorem find CO3-App (8)

$$Z^{-1}\left[\frac{10z^2}{(5z-2)(2z+1)}\right]$$

19. (a) Express $f(x) = \frac{1}{2}(\pi - x)$ as a Fourier series of period 2π in the interval $0 < x < 2\pi$. CO4-App (16)

Or

- (b) (i) The table of values of the function $y = f(x)$ is given below: CO4-App (8)

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y:	1.8	0.3	0.5	2.16	1.3	1.7	1.8

Find a Fourier series up to the third harmonic to represent $f(x)$ in terms of x .

- (ii) Find the half range sine series for $f(x) = x$ in $(0, \pi)$ CO4-App (8)

20. (a) Find the Fourier transform of CO5-App (16)

$$f(x) = \begin{cases} 1 - |x|, & \text{if } |x| \leq 1 \\ 0 & \text{if } |x| > 1 \end{cases} \quad \text{and hence deduce that}$$

$$i) \int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2} \quad ii) \int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt = \frac{\pi}{3}$$

Or

(b) (i) Find the Fourier sine & cosine transform of e^{-ax}

CO5–App (8)

(ii) Evaluate $\int_0^{\infty} \frac{dx}{(x^2 + 25)^2}$

CO5–App (8)