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**Question Paper Code:R2M06**

B.E./B.Tech. DEGREE EXAMINATION, APRIL / MAY 2025

Second Semester

Civil Engineering

R21UMA206-DIFFERENTIAL EQUATIONS, COMPLEX ANALYSIS & TRANSFORM  
TECHNIQUES

(Regulations R2021)

Duration: Three hours

Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1.  $\frac{1}{(D-m)^2} e^{mx} = \underline{\hspace{2cm}}$  CO6-U  
(a)  $x e^{mx}$  (b)  $x^2 e^{mx}$  (c)  $\frac{x^2}{2} e^{mx}$  (d)  $\frac{x^2}{m} e^{mx}$
2. The auxiliary equation of the equation  $x^2 \frac{d^2 y}{dx^2} + 4x \frac{dy}{dx} + 2y = e^x$  is  $\underline{\hspace{2cm}}$  CO6-U  
(a)  $Ae^x + Be^{\frac{x}{4}}$  (b)  $Ae^{-x} + Be^{5x}$  (c)  $(A+Bx)e^{2x}$  (d)  $Ae^x + Be^{4x}$
3. Divergence of vector  $x^2 \vec{i} + y^2 \vec{j} + z^2 \vec{k}$  at (1, 2, -3) is  $\underline{\hspace{2cm}}$  CO2- App  
(a) 8 (b) 4 (c) -3 (d) 0
4. If  $\phi = x^2 + y^2 - z - 10$  then  $|\nabla \phi|$  at (1, 1, 1) is  $\underline{\hspace{2cm}}$  CO2- App  
(a)  $2(\vec{i} + \vec{j} + \vec{k})$  (b)  $2\vec{i} + 2\vec{j} - \vec{k}$  (c) 3 (d) 9
5. The PDE obtained from  $z = (x+a)(y+b)$  is  $\underline{\hspace{2cm}}$ . CO3- App  
(a)  $3z = px + qy$  (b)  $py - qx = 0$  (c)  $z = pq$  (d)  $px + qy = 0$
6. The general solution of  $2r + 5s - 3t = 0$  is  $\underline{\hspace{2cm}}$  CO3- App  
(a)  $f_1(y + 3x) + f_2(y - \frac{x}{2})$  (b)  $f_1(3y + x) + f_2(y + 2x)$   
(c)  $f_1(y - 3x) + f_2(y + \frac{x}{2})$  (d)  $f_1(3y - x) + f_2(y - 2x)$

7. If  $f(z)$  is analytic at all points inside and on a simple closed curve  $c$ , then CO4- App  
 $\int_c f(z) dz = \text{-----}$   
 (a)  $2\pi i$  (b)  $-2\pi i$  (c)  $4\pi i$  (d) 0
8. The value of  $\int_c \frac{dz}{z+2}$ ,  $c : |z| = 1$  is CO4- App  
 (a)  $2\pi i$  (b)  $-2\pi i$  (c)  $4\pi i$  (d) 0
9.  $L(\sinh at) = \text{_____}$  CO6- U  
 (a)  $\frac{s}{s^2 - a^2}$  (b)  $\frac{a}{s^2 - a^2}$  (c)  $\frac{s}{s^2 + a^2}$  (d)  $\frac{a}{s^2 + a^2}$
10.  $L^{-1}(1) = \text{_____}$  CO5- Ap  
 (a)  $t$  (b)  $\frac{1}{t}$  (c)  $\frac{1}{s}$  (d)  $\delta(t)$

PART – B (5 x 2= 10 Marks)

11. Compute the particular integral of  $(D^2 + 5D + 6)y = e^{2x}$  CO1- App
12. Compute the Directional derivative of  $\phi = 4xz^2 + x^2yz$  at  $(1, -2, -1)$  in the CO2- App  
 direction  $2\vec{i} + 3\vec{j} + 4\vec{k}$ .
13. Form the PDE of all spheres whose centers lie on  $Z$ - axis CO3- App
14. Evaluate  $\int_c \frac{e^{-z}}{z+1} dz$  where  $C$  is  $|z| = \frac{1}{2}$  using Cauchy integral formula CO1 App
15. Compute  $L(e^{-2t})$  CO6 U

PART – C (5 x 16= 80 Marks)

16. (a) (i) Using method of variation of parameters solve  $(D^2 + 4)y = \sec 2x$ . CO1- App (8)  
 (ii) A colony of bacteria is growing exponentially. At time  $t=0$  it has 10 bacteria in it, and at time  $t=4$  it has 2000. At what time will it have 100,000 bacteria? CO1- App (8)
- Or
- (b) (i) Solve:  $(x^2 D^2 + xD + 1)y = x \sin(\log x)$  CO1- App (8)  
 (ii) Solve:  $(D^2 - 4D + 3)y = \sin 3x + e^{2x}$  CO1- App (8)

17. (a) Verify Green's theorem for  $\oint_C (x^2 + xy) dx + (x^2 + y^2) dy$  where  $C$  is the square bounded by the line  $x = 0, x = 1, y = 0$  &  $y = 1$ . CO2- App (16)
- Or
- (b) Verify Gauss Divergence theorem for  $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$  where  $S$  is the surface of the cuboid formed by the planes  $x = 0, x = a, y = 0, y = b, z = 0$  &  $z = c$ . CO2- App (16)
18. (a) (i) Solve:  $(mz - ny)p + (nx - lz)q = ly - mx$  CO3- App (8)
- (ii) Solve  $(D^2 - DD' - 3D'^2)z = \sin(x+y) + e^{6x+y}$  CO3- App (8)
- Or
- (b) (i) Solve  $(D^2 + DD' - 6D'^2)z = \cos(x+y) + e^{x+y}$  CO3- App (8)
- (ii) Solve  $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$  CO3- App (8)
19. (a) (i) Evaluate CO4- App (8)
- $$f(z) = \int_C \frac{\cos \pi z^2 + \sin \pi z^2}{(z-1)(z-2)} dz$$
- by using Cauchy's Integral formula
- where  $C$  is  $|z| = 3$
- (ii) Expand  $\frac{z-1}{(z+2)(z+3)}$  as Laurent's series valid in the region  $2 < |z| < 3$  CO4- App (8)
- Or
- (b) Using contour integration, to compute the value of  $\int_0^{2\pi} \frac{d\theta}{13+5\cos\theta}$  CO4- App (16)
20. (a) (i) Solve the differential equation  $\frac{d^2y}{dt^2} + y = \sin 2t$ ;  $y(0) = 0$ ;  $y'(0) = 0$  by using Laplace transform method. CO5- App (8)
- (ii) Compute the Laplace Transforms of  $\frac{\cos at - \cos bt}{t}$  CO5- App (8)
- Or

(b) (i) Using the periodic function, Compute the Laplace transform CO5- App (8)

of  $f(t) = \begin{cases} k, & 0 \leq t \leq a \\ -k, & a \leq t \leq 2a \end{cases}$

(ii) Using Convolution Theorem, Compute  $L^{-1} \left[ \frac{1}{(s^2 + a^2)(s^2 + b^2)} \right]$  CO5- App (8)