**A** **Reg. No. :** | | | | | | | | | | | | |

## Question Paper Code:R2M02

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2025

Second Semester

Mechanical Engineering

R21UMA202- CALCULUS, FOURIER SERIES AND NUMERICAL METHODS

(Regulations R2021)

(Common to Chemical Engineering)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. For any root the order of convergence of Newton's method is ________ CO6- U

   (a) 4 (b) 1 (c) 2 (d) 3

2. Gauss Jacobi iteration converges if the coefficient matrix is __________ dominant CO6- U

   (a) Squarely (b) logically (c) diagonally (d) symmetrically

3. The auxiliary equation of the equation $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$ is ____ CO2-App

   (a) $m^2$- 4 $m$ +5=0 (b) $m^2$+3 $m$ -2=0 (c) $m^2$+3m +2=0 (d) 2 $m^2$+5$m$ -7=0

4. The order and degree of $(y''')^2 + 2(y'')^3 + y = 0$ is _____ CO6-U

   (a) 3,2 (b) 2,3 (c) 3,3 (d) 2,2

5. Divergence of vector $x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ at (1, 2, -3) is _____ CO3-App

   (a) 4 (b) -4 (c) -3 (d) 0

6. If $\overline{F}$ is a conservative field, then $\overline{F}$ is_______ CO6-U

   (a) Solenoidal (b) Irrotational (c) 0 (d) None of these

7. The root mean square value of f(x) in (0, l) is --------- CO4- App

   (a) l (b) 1/2 (c) $l/\sqrt{3}$ (d) $2l$

8. If $f(-x) = -f(x)$, then f(x) is said to be an _____ CO6- U

(a) Odd Function (b) Even Function (c) Periodic function (d) Self Reciprocal

9. $F[xf(x)] =$ ______ CO6-U

(a) $-F_c[f(x)]$ (b) $-\frac{d}{ds}\{F_s[f(x)]\}$ (c) $-F_s[f(x)]$ (d) $-\frac{d}{ds}\{F_c[f(x)]\}$

10. In Modulation property, F[f(x) cosax] = ______ CO6-U

(a) $\frac{1}{2}[F(s+a)-F(s-a)]$ (b) $\frac{1}{2}[F(s+a)+F(s-a)]$

(c) $[F(s+a)-F(s-a)]$ (d) $F(s+a)+F(s-a)$

PART – B (5 x 2= 10 Marks)

11. Using Power method find the dominant Eigen value of CO1-App

$$\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$$

12. Find the particular integral of CO2-App

$(D^2+7D-8)y=e^{2x}$

13. Showthat $\nabla(r^n)=nr^{n-2}\vec{r}$. CO3- App

14. Find the root mean square value of the function f(x) = x in ( 0,L) CO4- App

15. Write Fourier cosine Transform pair. CO6- U

PART – C (5 x 16= 80 Marks)

16. (a) (i) Solve for a positive root of $3x - \cos x - 1 = 0$ by Newton's Raphson method CO1- App (8)

(ii) Solve $2x + 3y - z = 5$ , $4x + 4y -3z = 3$ , $2x -3y +2z =2$ by Gauss Elimination method CO1- App (8)

Or

(b) (i) Using Power method find numerically largest Eigen value of CO1- App (8)

$$\begin{pmatrix} 9 & 1 & 8 \\ 7 & 4 & 1 \\ 1 & 7 & 9 \end{pmatrix}$$

(ii) Solve $27x + 6y - z = 85$, $6x + 15y +2z = 72$, $x + y + 54z =110$ by Gauss Seidel method CO1 - App (8)

17. (a) (i) Using method of variation of parameters solve $(D^2 + a^2)y = \sec ax$ CO2-App (8)

(ii) Solve $(D^2 - D - 6)y = 3e^{4x} + 5$ CO2-App (8)

Or

(b) (i) Solve $(x^2D^2 - xD + 1)y = \left(\frac{\log x}{x}\right)^2$ CO2-App (8)

(ii) A colony of bacteria of growing exponentially. At time t=0 it has 10 bacteria in it and at time t = 4 it has 2000. At what time will it have 100,000 bacteria? CO2-App (8)

18. (a) Verify Gauss Divergence theorem for $\vec{F}=(x^2-yz)\vec{i}+(y^2-xz)\vec{j}+(z^2-xy)\vec{k}$ over the rectangular parallelepiped x = 0, x = a, y = 0, y = b, z = 0, z = c CO3-App (16)

Or

(b) (i) Prove that $\vec{F}=(x^2+xy^2)\vec{i}+(y^2+x^2y)\vec{j}$ is irrotational vector and find the Scalar potential such that $\bar{F} = \nabla\emptyset$. CO3-App (8)

(ii) Evaluate Stoke's theorem for $\int (x^2 - y^2)dx + 2xydy$ , where C is bounded by $x - 0, x = a, \quad y = 0 \quad and \quad y = b$ CO3-App (8)

19. (a) (i) Find the Fourier series of f(x) = $x^2$ in $(-\pi, \pi)$ CO4-App (8)

(ii) Find the Half range cosine series for $f(x) = x(\pi - x)$ in $(0, \pi)$. Deduce that $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}\ldots\ldots = \frac{\pi^4}{90}$ CO4 -App (8)

Or

(b) The table of values of the function $y = f(x)$ is given below: CO4- App (16)

| x | 0 | $\pi/3$ | $2\pi/3$ | $\pi$ | $4\pi/3$ | $5\pi/3$ | $2\pi$ |
|---|---|---|---|---|---|---|---|
| y: | 1.8 | 0.3 | 0.5 | 2.16 | 1.3 | 1.76 | 1.8 |

Find a Fourier series up to the third harmonic to represent f(x) in terms of x.

20. (a) Find the Fourier transform of $f(x) = \begin{cases} a - |x|, & \text{if } |x| \le a \\ 0 & \text{if } |x| > a \end{cases}$ and hence deduce that i) $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$ ii) $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt = \frac{\pi}{3}$ CO5-App (16)

Or

(b) (i) Evaluate CO5-App (8)

$$\int_0^\infty \frac{dx}{\left(x^2 + 9\right)^2}$$

(ii) Evaluate CO5-App (8)

$$\int_0^\infty \frac{x^2\,dx}{(x^2 + a^2)(x^2 + b^2)}$$