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Question Paper Code:R2M07

B.E./B.Tech. DEGREE EXAMINATION, APRIL / MAY 2025

Second Semester

Biomedical Engineering

R21UMA207- CALCULUS, COMPLEX ANALYSIS AND TRANSFORM
TECHNIQUES

(Regulations R2021)

(Common to Biotechnology and Agricultural Engineering branches)

Duration: Three hours

Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

- The complementary function of $(4D^2 - 3D - 1)y = 2 \sin 2x$ is CO1- App
(a) $Ae^x + Be^{\frac{x}{4}}$ (b) $Ae^{-x} + Be^{5x}$ (c) $(A+Bx)e^{2x}$ (d) $Ae^x + Be^{4x}$
- The order and degree of $(y''')^2 + 2(y'')^3 + y = 0$ is CO1- App
(a) 3,2 (b) 2,3 (c) 3,3 (d) 2,2
- Divergence of vector $x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ at (1, 2, -3) is CO2- App
(a) 8 (b) 4 (c) -3 (d) 0
- If $\vec{F}$ is Solenoidal then $\nabla \cdot \vec{F} =$ CO2- App
(a) 1 (b) 2 (c) 3 (d) 0
- The critical point of the transformation $w = z + \frac{1}{z}$ are CO3- App
(a) ± 1 (b) ± 2 (c) $\pm i$ (d) $-i$
- The mapping $w = z^2$ is not conformal at CO3- App
(a) 0 (b) -1 (c) 1 (d) 2
- The value of $\int_C \frac{dz}{z^2} = 0$ where C is CO4- App
(a) $|z| = 1$ (b) $|z - 1| = 2$ (c) $|z| = 2$ (d) $|z - 2| = 3$

8. The poles of $f(z) = \frac{z^2 + 1}{1 - z^2}$ are CO4- App
- (a) 1,0 (b) 0,0 (c) 1,2 (d) -1,1

9. $L(\sinh at) = \underline{\hspace{2cm}}$ CO6- U

- (a) $\frac{s}{s^2 - a^2}$ (b) $\frac{a}{s^2 - a^2}$ (c) $\frac{s}{s^2 + a^2}$ (d) $\frac{a}{s^2 + a^2}$

10. $L\left(\int_0^t e^{-t} dt\right) = \underline{\hspace{2cm}}$

- (a) $\frac{1}{s}$ (b) $\frac{1}{(s+1)}$ (c) $\frac{1}{s(s+1)}$ (d) $\frac{s}{(s+1)}$

PART – B (5 x 2= 10 Marks)

11. Calculate the Particular integral of $(D^2 + 3D + 2)y = \sin 3x$ CO1 App
12. Compute $\nabla\phi$, if $\phi = x^2 + y^2 + z^2$ at (1, -1, 1). CO2 App
13. Show that the function $f(z) = \bar{z}$ is nowhere differentiable. CO3 App
14. Using Cauchy's integral formula, Evaluate $\int_C \frac{z}{z-2} dz$ where C is $|z|=1$ CO4 App
15. Estimate $L[t \sin t]$. CO5 App

PART – C (5 x 16= 80 Marks)

16. (a) (i) Using method of variation of parameters solve $(D^2 + a^2)y = \operatorname{Cosec} ax$ CO1- App (8)
- (ii) A colony of bacteria is growing exponentially. At time $t=0$ it has 10 bacteria in it and at time $t=4$ it has 2000. At what time will it have 100,000 bacteria? CO1- App (8)

Or

- (b) (i) Solve $(D^2 + 2D + 2)y = e^{-2x} + \cos 2x$ CO1- App (8)
- (ii) Solve $(x^2 D^2 - xD + 1)y = \left(\frac{\log x}{x}\right)^2$ CO1- App (8)

17. (a) Verify Gauss Divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - xz)\vec{j} + (z^2 - xy)\vec{k}$ over the rectangular parallelepiped $x = 0, x = a, y = 0, y = b, z = 0, z = c$ CO2- App (16)
- Or
- (b) Verify Green's theorem for $\int_c (x^2 + y^2) dx - 2xy dy$ where c is the curve in the XY plane given by $x = \pm a, y = 0$, and $y = b$. CO2- App (16)
18. (a) (i) Determine the analytic function whose real part is $\frac{\sin 2x}{\cosh 2y - \cos 2x}$ CO3- App (8)
- (ii) Determine the bilinear transformation which maps $z = 1, i, -1$ respectively onto $w = i, 0, -i$ CO3- App (8)
- Or
- (b) (i) Determine the image of $|z - 2i| = 2$ under the transformation $w = \frac{1}{z}$ CO3- App (8)
- (ii) If $f(z) = u + iv$ is an analytic function then Prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4 |f'(z)|^2$ CO3- App (8)
19. (a) (i) Using Cauchy's integral formula, Evaluate $\int_c \frac{z+1}{(z-3)(z-1)} dz$ where C is the circle $|z| = 2$. CO4- App (8)
- (ii) Evaluate $f(z) = \frac{7z-2}{z(z+1)(z-2)}$ in Laurent's series valid in the region $1 < |z+1| < 3$ CO4- App (8)
- Or
- (b) Using Contour integration Prove that $\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + a^2)(x^2 + b^2)} dx = \frac{\pi}{a+b}, a > b > 0$ CO4- App (16)

20. (a) (i) Using Convolution Theorem, Compute

CO5- App (8)

$$\mathcal{L}^{-1} \left[\frac{s^2}{(s^2 + 9)(s^2 + 4)} \right]$$

(ii) Solve the differential equation $y'' - 3y' - 4y = 2e^{-t}$,
 $y(0) = 1$ & $y'(0) = 1$ by using Laplace transform method

CO5- App (8)

Or

(b) (i) Compute the Laplace transform of

CO5- App (8)

$$f(t) = \begin{cases} k, & 0 \leq t \leq a \\ -k, & a \leq t \leq 2a \end{cases}$$

(ii) Using Convolution Theorem, Compute $\mathcal{L}^{-1} \left[\frac{s^2}{(s^2 + a^2)(s^2 + b^2)} \right]$

CO5- App (8)