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Question Paper Code: R4M24

B.E./B.Tech. DEGREE EXAMINATION, APRIL / MAY 2025

Fourth Semester

Biomedical Engineering

R21UMA424- PROBABILITY AND INFERENTIAL STATISTICS

(Common to Biotechnology)

(Regulations R2021)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - $(10 \times 2 = 20 \text{ Marks})$

If the probability mass function of a random variable X is given by P(X = r) = Kr², r = 1, 2, 3, 4. Find k.
Define: Exponential Distribution and its mean, variance.

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3. Define: Rank Correlation CO2- App

4. Define: correlation coefficient between X and Y. CO2- App

5. State any two properties of an autocorrelation function. CO6- U

6. Show that autocorrelation function is an even function. CO3- App

7. What is unit impulse response of a system? CO6- U

8. Define white noise.

9. Write the application of 't' test.

10. Write the formula for the chi- square test of goodness of fit of a random CO6-U sample to a hypothetical distribution

$$PART - B (5 \times 16 = 80 \text{ Marks})$$

11. (a) (i) A R.V. X has the following distribution

 $CO1-App \qquad (8)$

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|------|---|----|----|----|----|----|----|
| P(X) | a | 2a | 3a | 5a | 6a | 6a | 8a |

(a) Find 'a' (b) Find $P(X \ge 2)$, $P(2 \le X \le 5)$ and (c) Find E(X).

(ii) A manufacturer produces light-bulbs that are packed into CO1-App (8) boxes of 100. If quality control studies indicate that 0.5% of the light-bulbs produced are defective, what percentage of the boxes will contain: (a) no defective? (b) 2 or more defectives?

Or

(b) (i) If the density function of a continuous r.v X is given by CO1- App (8)

$$f(x) = \begin{cases} ax & 0 \le x \le 1 \\ a & 1 \le x \le 2 \\ 3a - ax & 2 \le x \le 3 \\ 0 & otherwise \end{cases}$$
 (a) Find the value of "a"

- (b) Find the c.d.f. of X.
- (ii) Explain M.G.F of Exponential distribution and hence find CO1- App (8) mean and variance.
- 12. (a) (i) The joint probability mass function of (X,Y) is given by CO2 App (8) P(x,y) = k(2x+3y), x = 0, 1, 2; y = 1, 2, 3. Find all marginal distribution function and conditional distribution.
 - (ii) Joint pdf of x and y is $f(x,y) = \begin{cases} 2-x-y, 0 \le x, y \le 1 \\ 0 & elsewhere \end{cases}$. Find CO2 App (8)

marginal density function of x and y, mean of x, mean of y.s

Or

(b) Find coefficient of correlation between capital employed and CO 2 - App (16) profit obtained from the following data.

| Capital Employed (Rs. In Crore) | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
|---------------------------------|----|----|----|----|----|----|----|----|----|-----|
| Profit (Rs. In Crore) | | 4 | 8 | 5 | 10 | 15 | 14 | 20 | 22 | 50 |

And also find Regression Equations x on y & y on x.

13. (a) (i) Find power spectral densities of the following auto correlation CO3- App (8)

function
$$R(\tau) = e^{\frac{-\alpha^2 \tau}{2}}$$

(ii) Examine whether the random process $X(t) = A \cos(\alpha t + \theta)$ is CO3-App (8) a wide sense stationary if A and α are constants and θ is uniformly distributed random variable in $(0, 2\pi)$.

Or

- (b) Consider the two random process $X(t) = 5\cos[(\omega t + \theta)]$ and CO3-App (16) $Y(t) = 2\cos[(\omega t + \theta \frac{\pi}{2})]$. Where θ is uniformly distributed random variable in $(0, 2\pi)$ then Prove that $|R_{XY}(\tau)| \le \sqrt{R_{XX}(0)} \cdot R_{YY}(0)$.
- 14. (a) If X(t) is the input Voltage to a circuit and Y(t) is the Output CO4- App Voltage, X(t) is a stationary random process with E(x) = 0 and $R_{XX}(\tau) = e^{-4|\tau|}$. Find E(Y), $S_{XX}(\omega)$ & $S_{YY}(\omega)$ if the system transfer function is given by $|H(\omega)|^2 = \frac{1}{3^2 + \omega^2}$.

Oı

- (b) If the input to a time invariant stable linear system is a wide sense CO4- App stationary process. Prove that the output will also be a wide sense stationary process
- 15. (a) (i) 4 coins were tossed 160 times and the following results were CO5–App obtained:

| No. of heads: | 0 | 1 | 2 | 3 | 4 |
|-----------------------|----|----|----|----|---|
| Observed frequencies: | 17 | 25 | 27 | 33 | 6 |

Under the assumption that the coins are unbiased, find the expected frequencies of getting 0, 1, 2, 3, 4 heads and test the goodness of fit.

(ii) Two sample polls of votes for two candidates A and B for a CO5–App public office are taken one from among residents of rural areas. The results are given below. Examine whether the nature of the area is related to voting preference in this election.

| Area/Votes for | A | В | Total |
|----------------|------|-----|-------|
| Rural | 620 | 380 | 1000 |
| Urban | 550 | 450 | 1000 |
| Total | 1170 | 830 | 2000 |

Or

(b) (i) Two independent samples of sizes 9 and 7 from a normal CO5–App population had the following values of the variables.

| Sample I | 18 | 13 | 12 | 15 | 12 | 14 | 16 | 14 | 15 |
|--------------|----|----|----|----|----|----|----|----|----|
| Sample II | 16 | 19 | 13 | 16 | 18 | 13 | 15 | - | - |

Investigate the estimates of the population variance differ significantly at 5% level?

(ii) Two independent samples are chosen from two schools A and CO5-App (8) B, a common test is given in a subject. The scores of the students as follows:

| Section A | 76 | 68 | 70 | 43 | 94 | 68 | 33 | - |
|-----------|----|----|----|----|----|----|----|----|
| Section B | 40 | 48 | 92 | 85 | 70 | 76 | 68 | 22 |

Can we conclude that students of school A performed better than students of section B?