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**Question Paper Code: U4M24**

B.E./B.Tech. DEGREE EXAMINATION, MAY 2025

Fourth Semester

Bio Medical Engineering

21UMA424 - PROBABILITY AND INFERENCE STATISTICS

(Common to Bio Technology)

(Regulations 2021)

Duration: Three hours

Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. Probability of sure event is CO6- U  
(a) 0 (b) 1 (c) 2 (d) 10
2. For Binomial distribution mean is 6 and variance is 2 find  $P[X=x]$  CO1- App  
(a)  ${}^9C_x \frac{2^x}{3} \frac{1}{3} \frac{1}{3} \dots \frac{1}{3}$  (b)  ${}^6C_x \frac{1^x}{3} \frac{2^{n-x}}{3}$  (c)  ${}^{2C}_x \frac{1^x}{2} \frac{1^{n-x}}{2}$  (d) none of the above
3. If X and Y are independent then correlation coefficient between X and Y is CO6- U  
(a) 1 (b) 0 (c) -1 (d) 10
4. The regression lines of two random variables X, Y is given by CO2- Ana  
 $3x + y = 10$ ,  $3x + 4y = 12$ . The correlation between X, Y is  
(a) 1/2 (b) -1/2 (c) -1 (d) 1
5. Given  $R(\tau) = 25 + \frac{4}{1 + 6\tau^2}$  What is  $E[X^2(t)]$ ? CO3- App  
(a) 25 (b) 29 (c) 26 (d) 27
6. If the Random Process  $\{X(t)\}$  with mean has Auto correlation function CO3- App  
 $R(\tau) = 16 + 9e^{-|\tau|}$  then the mean value is  
(a) 16 (b) 25 (c) 0 (d) 4
7. If  $X(t)$  is the white noise process, then its power spectrum is given by CO6- U  
(a)  $S(\omega) = \frac{N_0}{4\pi}$  (b)  $S(\omega) = \frac{N_0}{4}$  (c)  $S(\omega) = \frac{N_0}{2} \delta(\tau)$  (d)  $S(\omega) = \frac{N_0}{2}$
8. The average power of the auto correlation function is  $R_{xx}(\tau) = 3e^{-3|\tau|}$  CO4- App  
(a) 3 (b) 6 (c) 2 (d) 0

9. The degrees of freedom for the sample size  $n=25$  in t test is \_\_\_\_\_. CO6- U  
 (a) 20 (b) 22 (c) 24 (d) 26
10. F-test is used to test for equality of \_\_\_\_\_. CO6- U  
 (a) Sample Mean (b) Variance (c) Population mean (d) All the above

PART – B (5 x 2= 10Marks)

11. A coin is tossed twice; find the probability that there will appear atleast one head? CO1-App
12. Find the value of K, if  $f(x, y) = Kxy$  in  $0 < x, y < 1$  is to be the joint density function CO2-Ana
13. The power spectrum of a WSS process  $X(t)$  is given by  $S_{xx}(\omega) = \frac{4}{4 + \omega^2}$  Find the autocorrelation. CO3-App
14. Calculate the value of the system transfer function, if the input of the system with impulse response  $h(t) = e^{-3t}U(t)$ . CO4-App
15. A sample of size 10 has mean 58, standard deviation 18.4 and population mean 50, Compute the calculated value of 't' distribution. CO5-Ana

PART – C (5 x 16= 80Marks)

16. (a) (i) A Random Variable  $X$  has the following probability distribution CO1-App (8)

|          |   |   |    |    |    |       |        |          |
|----------|---|---|----|----|----|-------|--------|----------|
| $X=x$    | 0 | 1 | 2  | 3  | 4  | 5     | 6      | 7        |
| $P(X=x)$ | 0 | k | 2k | 2k | 3k | $k^2$ | $2k^2$ | $7k^2+k$ |

Find (i) 'k'

(ii)  $P(X > 6)$ ,  $P(0 < X < 4)$

- (ii) Define Binomial distribution. Find the moment generating function and Hence find mean and variance. CO1-App (8)

Or

- (b) (i) If  $f(x) = \begin{cases} \frac{k}{1+x^2}, & -\infty < x < \infty \\ 0, & elsewhere \end{cases}$  is the Probability Density Function CO1-App (8)

of a Random variable  $X$ ,

(i) Find K (ii) distribution function of  $F(x)$ .

- (ii) State and Prove the memoryless property for an Exponential Property CO1-App (8)

17. (a) From the following data, find (i) the two regression equations CO2-Ana (16)  
(ii) the coefficient of correlation between the marks in Economics and Statistics (iii) the most likely marks in Statistics when marks in Economics are 30

|                     |    |    |    |    |    |    |    |    |    |    |
|---------------------|----|----|----|----|----|----|----|----|----|----|
| Marks in Economics  | 25 | 28 | 35 | 32 | 31 | 36 | 29 | 38 | 34 | 32 |
| Marks in Statistics | 43 | 46 | 49 | 41 | 36 | 32 | 31 | 30 | 33 | 39 |

Or

- (b) (i) Obtain the Correlation coefficient for the following heights (in inches) of fathers X and their sons Y. CO2-Ana (8)

|   |    |    |    |    |    |    |    |    |
|---|----|----|----|----|----|----|----|----|
| X | 65 | 66 | 67 | 67 | 68 | 60 | 70 | 72 |
| Y | 67 | 68 | 65 | 68 | 72 | 72 | 69 | 71 |

- (ii) If the probability density function of a two dimensional random variable X and Y is given by  $f(x, y) = (x + y)$ ,  $0 < x, y < 1$  CO2-Ana (8)

Find the probability density function of  $U = XY$ .

18. (a) (i) If the auto correlation function of the random binary transmission is given by CO3-App (8)

$$R_{xx}(\tau) = \begin{cases} 1 - \frac{|\tau|}{T} & ; |\tau| \leq T \\ 0 & ; |\tau| \geq T \end{cases}$$

Find the Power spectral density function.

- (ii) A stationary process has an autocorrelation function given by CO3-App (8)

$$R(\tau) = 25 + \frac{4}{1 + 6\tau^2}$$

Find the Mean and Variance

Or

- (b) (i) If the Power spectral density of a WSS processes is given by CO3-App (8)

$$S(\omega) = \begin{cases} \frac{b}{a}(a - |\omega|) & ; |\omega| \leq a \\ 0 & ; |\omega| > a \end{cases}$$

Find the auto correlation function of the Process

- (ii) Find power spectral densities of the following auto correlation CO3-App (8)

$$\text{function } R(\tau) = e^{-\frac{\alpha^2 \tau^2}{2}}$$

19. (a) A random process  $X(t)$  having the autocorrelation function  $R_{XX}(\tau) = P e^{-b|\tau|}$  Where  $b$  is a constant is applied to the input of the system with impulse response  $h(t) = e^{-bt}U(t)$  where  $b$  is a constant. Find the autocorrelation of the output  $Y(t)$ . CO4-App (16)

Or

- (b) If  $X(t)$  is a WSS process and if CO4-App (16)

$$Y(t) = \int_{-\infty}^{\infty} h(u) X(t-u) du \text{ then}$$

$$(i). R_{XY}(\tau) = R_{XX}(\tau) * h(\tau) \quad (ii). R_{YY}(\tau) = R_{XX}(\tau) * h(-\tau)$$

$$(iii). S_{XY}(\omega) = S_{XX}(\omega) * H(\omega) \quad (iv). S_{YY}(\omega) = S_{XX}(\omega) * |H(\omega)|^2$$

20. (a) Two researchers A and B adopted different techniques while rating the student's level. Identify the Sampling distribution; Can you say that the techniques adopted by them are significant? CO5-Ana (16)

| Researchers | Below Average | Average | Above Average | Genius | Total |
|-------------|---------------|---------|---------------|--------|-------|
| A           | 40            | 33      | 25            | 2      | 100   |
| B           | 86            | 60      | 44            | 10     | 200   |
| Total       | 126           | 93      | 69            | 12     | 300   |

Or

- (b) Two independent samples of sizes 9 and 7 from a normal population had the following values of the variables. CO5-Ana (16)

|           |    |    |    |    |    |    |    |    |    |
|-----------|----|----|----|----|----|----|----|----|----|
| Sample I  | 18 | 13 | 12 | 15 | 12 | 14 | 16 | 14 | 15 |
| Sample II | 16 | 19 | 13 | 16 | 18 | 13 | 15 |    |    |

Identify the sampling distribution, Do the estimates of the population variance differ significantly.