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Question Paper Code: R2M11

B.E./B.Tech. DEGREE EXAMINATION, APRIL / MAY 2025

Second Semester

CSE (Artificial Intelligence and Machine Learning)

R21UMA211-FOURIER SERIES, PARTIAL DIFFERENTIAL EQUATIONS AND
COMPLEX ANALYSIS

(Regulations R2021)

Duration: Three hours

Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The root mean square value of $f(x) = x$ in $(0, l)$ is ----- CO1-App
(a) l (b) $l/2$ (c) $l/\sqrt{3}$ (d) $2l$
2. If $f(-x) = -f(x)$, then $f(x)$ is said to be an _____ CO6- U
(a) Odd Function (b) Even Function (c) Periodic function (d) Self Reciprocal
3. The complimentary function of $(D^2 - 2DD' + D'^2)z = 0$ is _____. CO2-App
(a) $f_1(y+x) + f_2(y-x)$ (b) $f_1(y+x) + x f_2(y+x)$
(c) $f_1(y+x) + x^2 f_2(y+x)$ (d) None of these
4. The subsidiary equations of Lagrange's linear equation is ----- CO6 - U
(a) $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ (b) $\frac{dx}{P} + \frac{dy}{Q} + \frac{dz}{R}$ (c) $\frac{dx}{P} - \frac{dy}{Q} - \frac{dz}{R}$ (d) $Pp + Qq = R$
5. Classify the equation $u_{xx} + u_{yy} = 0$ is _____ CO3-App
(a) parabolic (b) hyperbolic (c) elliptic (d) cyclic
6. In a one dimensional wave equation, $c^2 =$ _____. CO6- U
(a) T^2/m^2 (b) T/m (c) T/m^2 (d) T^2/m
7. The real and imaginary parts of an analytic function are _____ CO6- U
(a) Harmonic (b) orthogonal
(c) satisfies Laplace equation (d) All the above are true

8. The mapping $w = z^2$ is not conformal at _____ CO6- U
 (a) 0 (b) -1 (c) 1 (d) 2
9. Simple pole is a pole of order _____ CO6- U
 (a) 1 (b) 2 (c) 3 (d) 4
10. The value of $\int_C \frac{dz}{z^2} = 0$ where C is _____ CO6- U
 (a) $|z| = 1$ (b) $|z - 1| = 2$ (c) $|z| = 2$ (d) $|z - 2| = 1$

PART – B (5 x 2= 10 Marks)

11. Find a_0 and a_n in the Fourier series of $f(x) = x$ in $(0, 2\pi)$ CO1 App
12. Form the PDE by eliminating the constant f from $z = f(x^2 - y^2)$ CO2 App
13. The ends A and B of a rod 1 cm long have the temperatures 40°C and 90°C until steady state condition prevails. Find the subsequent temperature distribution in the rod. CO3 App
14. Find the constant 'a' so that $u(x, y) = ax^2 - y^2 + xy$ is harmonic CO4 App
15. Expand $\frac{1}{z(z-1)}$ as a Laurent series about $z=0$ in the annulus $0 < |z| < 1$. CO5 App

PART – C (5 x 16= 80 Marks)

16. (a) (i) Express $f(x) = (\pi - x)^2$ as a Fourier series of period 2π in the interval $0 < x < 2\pi$. CO1- App (8)

(ii) The table of values of the function $y = f(x)$ is given below: CO1- App (8)

x:	0	T/6	T/3	T/2	2T/3	5T/6	T
y:	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Find a Fourier series upto the third harmonic to represent $f(x)$ in terms of x

Or

- (b) (i) Compute first two harmonics of the Fourier series for the following data. CO1- App (8)

x	0	2	4	6	8	10
y	1.87	1.36	1.14	1.05	-0.83	-0.36

(ii) Find the Half range cosine series for $f(x) = x$ in $(0, \pi)$ CO1- App (8)

17. (a) (i) Solve $(D^2 - DD' - 2D'^2)Z = e^{3x+4y} + \cos(x+y)$ CO2- App (8)
- (ii) Solve $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$ CO2- App (8)
- Or
- (b) (i) Solve $Z = px + qy + p^2 - q^2$ CO2- App (8)
- (ii) Form a PDE by eliminating arbitrary functions from $\phi(x^2 + y^2 + z^2, x + y + z) = 0$. CO2- App (8)
18. (a) A bar of 10cm long with insulated sides has its ends A and B kept at 50°C and 100°C respectively. Until steady state condition prevails. The temperature at A is then suddenly raised to 90°C and at the same instant B is lowered to 60°C and maintained thereafter. Find the subsequent temperature distribution in the bar. CO3- App (16)
- Or
- (b) If a string of length 'l' is initially at rest in its equilibrium position and each of its points is given velocity, $\frac{\partial y}{\partial t} = v_0 \sin^3 \frac{\pi x}{l}, 0 < x < l$, Determine the displacement function $y(x, t)$. CO3- App (16)
19. (a) (i) Find the image of $|z - 3i| = 3$ under the transformation $w = \frac{1}{z}$ CO4- App (8)
- (ii) If $f(z) = u + iv$ is a regular function of z in a domain D the following relation hold in D . $\nabla^2 |f(z)|^2 = 4 |f'(z)|^2$ CO4- App (8)
- Or
- (b) (i) Find the bilinear transformation which maps the points 1, i, -1 on to 0, 1, ∞ . Prove that the transformation maps the interior of the unit circle of Z -plane onto upper half of the w -plane CO4- App (8)
- (ii) Find the image of infinite strips CO4- App (8)
- (i) $\frac{1}{4} < y < \frac{1}{2}$ (ii) $0 < y < \frac{1}{2}$ under the transformation $w = \frac{1}{z}$

20. (a) (i) Evaluate using Cauchy's Residue theorem for CO5- App (8)

$$f(z) = \int_C \frac{3z^2 + z - 1}{(z^2 - 1)(z - 3)} dz, \text{ where 'C' is the circle } |z| = 2.$$

- (ii) Evaluate $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent series valid for the CO5- App (8)

region

$$1 < |z| < 3.$$

Or

- (b) (i) Evaluate CO5- App (8)

$$f(z) = \int_C \frac{\cos \pi z^2 + \sin \pi z^2}{(z - 1)(z - 2)} dz \text{ by using Cauchy Residue theorem}$$

where C is $|z| = 3$.

- (ii) Expand $\frac{1}{z(z - 1)}$ as Laurent's series valid in the regions CO5- App (8)

$$0 < |z| < 1$$