A **Reg. No. :** | | | | | | | | | | | | |

Question Paper Code: R2M02

B.E./B.Tech. DEGREE EXAMINATION, MAY 2024

Second Semester

Mechanical Engineering

R21UMA202 - CALCULUS, FOURIER SERIES AND NUMERICAL METHODS

(Regulations R2021)

(Common to Chemical Engineering)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. For any root the order of convergence of Newton's method is ______ CO6- U

   (a) 4 (b) 1 (c) 2 (d) 3

2. Gauss Seidel method converges faster than ___________ CO6- U

   (a) Gauss Elimination (b) Gauss Jacobi (c) Gauss Jordan (d) Newton's

3. $\frac{1}{(D-m)^2}e^{mx}$ = _____ CO2- App

   (a) xe^{mx} (b) x^2e^{mx} (c) $\frac{x^2}{2}e^{mx}$ (d) $\frac{x^2}{m}e^{mx}$

4. One of the solutions $y'' + 4y' + 4y = 0$ is ______ CO2- App

   (a) xe^{-2x} (b) xe^{2x} (c) x^2e^{2x} (d) e^{2x}

5. Divergence of vector $x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ at (1, 2, -3) is _____ CO3- App

   (a) 4 (b) 4 (c) -3 (d) 0

6. $r = |\vec{r}|$ then ∇r^n = _____ CO3- App

   (a) $nr^{n-2}\vec{r}$ (b) $r^{n-2}\vec{r}$ (c) $\vec{i} + \vec{j} + \vec{k}$ (d) 0

7. If a function f(x) is even, its Fourier expansion contains only ___terms CO6- U

   (a) Sine (b) Cosine (c) tan (d) None of these

8. The root mean square value of f(x) in (0, l) is --------- CO4- App

   (a) l (b) 1/2 (c) $l/\sqrt{3}$ (d) $2l$

9. Convolution theorem on Fourier Transform is F[f(x)*g(x)]= ______ CO6-U

(a) F(s).G(s) (b) f(s).g(s) (c) F(s)*G(s) (d) f(s)*g(s)

10. $F_s[e^{-ax}] =$ _______ CO5-AP

(a) $\sqrt{\frac{2}{\pi}}\frac{s}{s^2+a^2}$ (b) $\sqrt{\frac{2}{\pi}}\frac{a}{s^2+a^2}$ (c) $\sqrt{\frac{2}{\pi}}\frac{a^2}{s^2+a^2}$ (d) $\sqrt{\frac{2}{\pi}}\frac{s^2}{s^2+a^2}$

PART – B (5 x 2= 10 Marks)

11. Compare Gauss Elimination and Gauss Jordan Methods CO6- U

12. Solve CO2 App

$$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$$

13. Find $\nabla\varphi$, if $\varphi = x^2 + y^2 + z^2$ at (1, -1, 1). CO3 App

14. Explain why tan x cannot be expanded in Fourier series CO6- U

15. *Calculate* $F_c\left(e^{-ax}\right)$ CO5 App

PART – C (5 x 16= 80Marks)

16. (a) (i) Solve 4x + 2y +z = 14, x + 5y -z = 10, x + y + 8z =20 by Gauss Elimination method CO1 App (8)

(ii) Solve for a positive root of x log x - 1.2=0 by Newton's Raphson method . CO1 App (8)

Or

(b) (i) Using Newton's Raphson method find the real positive root of $x^4 - x - 10 = 0$ CO1 App (8)

(ii) Solve 4x + 2y +z = 14, x + 5y -z = 10, x + y + 8z =20 by Gauss Elimination method CO1 App (8)

17. (a) (i) Using method of variation of parameters solve $(D^2 + a^2)y = \sec ax$ CO2 App (8)

(ii) Solve $(D^2 - D - 6)y = 3e^{4x} + 5$ CO2 App (8)

Or

(b) (i) Solve $(D^2 - 3D + 2)y = 2e^x + 2\cos 2x$ CO2 App (8)

(ii) If the population of a country double in 50 years , in how many years will it triple under the assumption that the rate of increase of proportional to the number of inhabitants? CO2 App (8)

18. (a) Verify Gauss divergence theorem for the vector function (16)
$\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ over the cube bounded by CO3 App
$x = 0, y = 0, z = 0$ *and* $x = 1, y = 1, z = 1$.

Or

(b) (i) Prove that $\vec{F} = (x^2 + xy^2)\vec{i} + (y^2 + x^2 y)\vec{j}$ is irrotational vector and find the Scalar potential such that $\bar{F} = \nabla\emptyset$. CO3 App (8)

(ii) Evaluate Stoke's theorem for $\int (x^2 - y^2)dx + 2xydy$, where C is bounded by $x - 0, x = a, \quad y = 0 \quad and \quad y = b$ CO3 App (8)

19. (a) (i) Express $f(x) = x^2$ as a Fourier series of period 2π in the interval $0 < x < 2\pi$. CO4App (8)

(ii) The table of values of the function $y = f(x)$ is given below: CO4 App (8)

x:	0	1	2	3	4	5
Y:	4	8	15	7	6	2

Find a Fourier series up to the third harmonic to represent f(x)in terms of x

Or

(b) (i) Express $f(x) = \frac{1}{2}(\pi - x)$ as a Fourier series of period 2π in the internal $0 < x < 2\pi$. CO4 App (8)

(ii) The table of values of the function $y = f(x)$ is given below: CO4App (8)

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y:	1.8	0.3	0.5	2.6	1.3	1.7	1.8

Find a Fourier series up to the third harmonic to represent f(x) in terms of x.

20. (a) (i) Prove that CO5 App (8)

$f(x) = e^{\frac{-x^2}{2}}$ is self-reciprocal under Fourier transform.

(ii) Evaluate CO5 App (8)

$$\int_0^\infty \frac{x^2}{(x^2 + a^2)^2}$$

Or

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(b) Show that the Fourier transform of CO5 App (16)

$$f(x)=\begin{cases} a^2-x^2 & |x|<a \\ 0 & |x|>a>0 \end{cases}$$ is $2\sqrt{\frac{2}{\pi}}\left[\frac{\sin sa - sa\cos sa}{s^3}\right]$. Hence deduce

$$\int_0^\infty \frac{\sin t - t\cos t}{t^3}\,dt = \pi/4$$ and $$\int_0^\infty \left(\frac{\sin t - t\cos t}{t^3}\right)^2 dt = \pi/15.$$