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**Reg. No. :**

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**Question Paper Code: U2M02**

B.E./B.Tech. DEGREE EXAMINATION, MAY 2024

Second Semester

Mechanical Engineering

21UMA202 - Calculus, Fourier Series And Numerical Methods

(Regulations 2021)

Duration: Three hours

Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

- Gauss Jacobi iteration converges if the coefficient matrix is \_\_\_\_\_ dominant. CO6-U  
(a) Squarely (b) logically (c) diagonally (d) symmetrically
- Newton's method also called \_\_\_\_\_ method. CO6-U  
(a) tangents (b) slope (c) secants (d) false
- The complementary function of  $(4D^2 - 3D - 1)y = 2 \sin 2x$  is \_\_\_\_\_. CO2-App  
(a)  $Ae^x + Be^{\frac{x}{4}}$  (b)  $Ae^{-x} + Be^{5x}$  (c)  $(A+Bx)e^{2x}$  (d)  $Ae^x + Be^{4x}$
- $\frac{1}{D^2}(\cos x) =$  \_\_\_\_\_. CO2-App  
(a)  $\sin x$  (b)  $-\cos x$  (c)  $\cos x$  (d)  $\tan x$
- Divergence of vector  $x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$  at (1, 2, -3) is \_\_\_\_\_. CO3- App  
(a) 4 (b) -4 (c) -3 (d) 0
- If  $\phi = x^2 + y^2 - z - 10$  then  $|\nabla \phi|$  at (1, 1, 1) is \_\_\_\_\_. CO3- App  
(a)  $2\vec{i} + 2\vec{j} + \vec{k}$  (b)  $2\vec{i} + 2\vec{j} - \vec{k}$  (c) 3 (d) 9
- $\cos x$  is a periodic function with period ----- CO4-App  
(a)  $\pi$  (b)  $2\pi$  (c)  $\pi/3$  (d)  $2\pi/3$

8. If  $f(-x) = -f(x)$ , then  $f(x)$  is said to be an \_\_\_\_\_. CO6-U  
 (a) Odd Function      (b) Even Function      (c) Periodic function      (d) Self Reciprocal
9. Convolution theorem on Fourier Transform is  $F[f(x)*g(x)]=$  \_\_\_\_\_ CO6-U  
 (a)  $F(s).G(s)$       (b)  $f(s).g(s)$       (c)  $F(s)*G(s)$       (d)  $f(s)*g(s)$
10.  $F_s[e^{-ax}] =$  \_\_\_\_\_ CO5-U  
 (a)  $\sqrt{\frac{2}{\pi}} \frac{s}{s^2+a^2}$       (b)  $\sqrt{\frac{2}{\pi}} \frac{a}{s^2+a^2}$       (c)  $\sqrt{\frac{2}{\pi}} \frac{a^2}{s^2+a^2}$       (d)  $\sqrt{\frac{2}{\pi}} \frac{s^2}{s^2+a^2}$

PART – B (5 x 2= 10Marks)

11. State the principle used in Gauss Elimination Method. CO6-U
12. Find the complete solution of  $(D^3 - 6D^2 + 11D - 6)y = 0$  CO2-App
13. Show that  $\nabla (r^n) = nr^{n-2}\vec{r}$ . CO3-App
14. Find  $b_n$  in the Fourier series of  $f(x) = |\cos x|$  in  $(0, 2\pi)$ . CO4-App
15. Write the Fourier sine transforms pair and Fourier Cosine transform pair. CO5-App

PART – C (5 x 16= 80Marks)

16. (a) (i) Solve for a positive root of  $3x - \cos x - 1=0$  by Newton's Raphson method. CO1-App (8)  
 (ii) Solve  $4x + 2y + z = 14$ ,  $x + 5y - z = 10$ ,  $x + y + 8z = 20$  by Gauss Elimination method CO1- App (8)
- Or
- (b) (i) Using Power method find numerically largest Eigen value of CO1- App (8)  

$$\begin{pmatrix} 9 & 1 & 8 \\ 7 & 4 & 1 \\ 1 & 7 & 9 \end{pmatrix}$$
- (ii) Solve  $27x + 6y - z = 85$ ,  $6x + 15y + 2z = 72$ ,  $x + y + 54z = 110$  by Gauss Seidel method CO1- App (8)
17. (a) (i) Using method of variation of parameters solve CO2-App (8)  
 $(D^2 + a^2)y = \operatorname{cosec} ax$   
 (ii) Solve  $(D^2 + 2D + 2)y = \cos 2x$  CO2-App (8)

Or

- (b) (i) Solve  $(D^2 - 3D + 2)y = 2e^x + 2\cos 2x$  CO2 -App (8)
- (ii) If the population of a country double in 50 years, in how many years will it triple under the assumption that the rate of increase of proportional to the number of inhabitants? CO2 -App (8)
18. (a) Verify Gauss Divergence theorem for  $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - xz)\vec{j} + (z^2 - xy)\vec{k}$  over the rectangular parallelepiped  $x = 0, x = a, y = 0, y = b, z = 0, z = c$  CO3-App (16)
- Or
- (b) (i) Prove that  $\vec{F} = (x^2 + xy^2)\vec{i} + (y^2 + x^2y)\vec{j}$  is irrotational vector and find the Scalar potential such that  $\vec{F} = \nabla\phi$ . CO3-App (8)
- (ii) Evaluate Green's theorem for  $\int (x^2 - y^2)dx + 2xydy$ , where C is bounded by  $x = 0, x = a, y = 0$  and  $y = b$  CO3-App (8)
19. (a) Find the Fourier series of  $f(x) = |\sin x|$  in  $-\pi < x < \pi$  of periodicity  $2\pi$ . CO4-App (16)
- Or
- (b) (i) The table of values of the function  $y = f(x)$  is given below: CO4-App (8)
- |    |   |   |    |   |   |   |
|----|---|---|----|---|---|---|
| x: | 0 | 1 | 2  | 3 | 4 | 5 |
| Y: | 4 | 8 | 15 | 7 | 6 | 2 |
- Find a Fourier series up to the third harmonic to represent  $f(x)$  in terms of  $x$ .
- (ii) Find the half range sine series for  $f(x) = x^2$  in  $(0, l)$  CO4-App (8)
20. (a) Find the Fourier transform of  $f(x) = \begin{cases} 1 - |x|, & \text{if } |x| \leq 1 \\ 0 & \text{if } |x| > 1 \end{cases}$  and hence CO5-App (16)
- deduce that i)  $\int_0^{\infty} \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$ .
- Or
- (b) (i) Find Fourier Cosine and Sine Transform of  $e^{-ax}$ . CO5- App (8)
- (ii) Evaluate  $\int_0^{\infty} \frac{x^2 dx}{(x^2 + a^2)(x^2 + b^2)}$  CO5- App (8)

