A **Reg. No. :** | | | | | | | | | | |

Question Paper Code: 51002

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2024

First Semester

Civil Engineering

15UMA102- ENGINEERING MATHEMATICS-I

(Common to ALL branches)

(Regulation 2015)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. $\lim_{x \to 1} \frac{x^2 - 1}{x - 1} =$ CO1-R

(a) 0 (b) 1 (c) 2 (d) 3

2. Suppose $f(x) = \begin{cases} \frac{x^2 - x}{2x} & if\ x \neq 0 \\ k & if\ x = 0 \end{cases}$. If $f(x)$ is continuous at x=0, then the value of 'k' is CO1-R

(a) -1 (b) 1 (c) $-\frac{1}{2}$ (d) $\frac{1}{2}$

3. If $u = (x-y)(y-z)(z-x)$ then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} =$ CO2-R

(a) 1 (b) 0 (c) 3 (d) 6

4. If $x = r\cos\theta, y = r\sin\theta$ *find* $\frac{\partial(x,y)}{\partial(r,\theta)}$ CO2-R

(a) r (b) r^2 (c) 0 (d) 2

5. $\Gamma\left(\frac{1}{2}\right) =$ CO3-R

(a) 1 (b) $\frac{1}{2}$ (c) $\sqrt{\pi}$ (d) π

6. $\int_0^{\frac{\pi}{2}} \cos^8 x\,dx =$ CO3-R

(a) $\frac{35\pi}{256}$ (b) $\frac{256}{35\pi}$ (c) 35π (d) 256

7. Value of the double integral $\int_0^1 \int_0^y dydx$ is CO4-R

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) $\frac{3}{4}$

8. $\int_0^{\frac{\pi}{2}} \int_0^{\sin\theta} r\,d\theta dr =$ CO4-R

(a) $\frac{1}{8}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{8}$ (d) π

9. The product of two eigen values of $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ is 16 then the third eigen value is CO5-R

(a) 3 (b) 2 (c) 4 (d) 5

10. If the product of two eigenvalues of third order singular matrix A is 34, then the third eigenvalue of the matrix A is CO5-R

(a) 3 (b) -1 (c) 1 (d) 0

PART – B (5 x 2= 10Marks)

11. Find CO1-R

$\frac{dy}{dx}$ given $x = acos^3\theta;\ y = asin^3\theta$.

12. State Euler's theorem on homogeneous functions. CO2-U

13. Evaluate $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^2 x\,dx$ CO3-R

14. Prove that $\int_{-1}^{3} \int_{x^2}^{2+x} dydx = \frac{8}{3}$. CO4-R

15. State Cayley Hamilton theorem. CO5-U

PART – C (5 x 16= 80Marks)

16. (a) (i) Find $\frac{dy}{dx}$ if $y = \tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}$ CO1-App (16)

Or

(b) (i) If $y = e^{ax}\sin bx,$ prove that $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + \left(a^2 + b^2\right)y = 0$ CO1-App (8)

(ii) Obtain the Maclaurin's series for $\log(1+x)$ CO1-App (8)

17. (a) (i) Verify Euler's theorem for the function $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$ CO2-App (8)

(ii) Given the transformations $u = e^x \cos y \, and \, v = e^x \sin y$ and that ϕ is a function of u and v and also of x and y, prove that $\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = (u^2 + v^2)\left[\frac{\partial^2\phi}{\partial u^2} + \frac{\partial^2\phi}{\partial v^2}\right]$ CO2-App (8)

Or

(b) Given the transformation $u = e^x \cos y \; and \; v = e^x \sin y$ and that f is a function of u and v and also of x and y, prove that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = (u^2 + v^2)\left(\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2}\right)$ CO2-App (16)

18. (a) (i) Evaluate $\int \frac{1}{1+\cos x}dx$ CO3-App (8)

(ii) Evaluate $\int e^{2\log x} e^{x^3} dx$ CO3-App (8)

Or

(b) Prove that CO3-App (16)

$\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

19.	(a)	Change the order of integration $\int_0^1 \int_{y^2}^{2-y} xy\,dy\,dx$ and hence evaluate it	CO4-App	(16)

Or

	(b)	(i) Find the smaller area bounded by the parabolas $y^2 = 4 - x$ and $y^2 = 4 - 4x$.	CO4-App	(8)
		(ii) Find the volume of the Sphere $x^2 + y^2 + z^2 = a^2$.	CO4-App	(8)
20.	(a)	(i) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$	CO5-App	(8)
		(ii) Using Cayley Hamilton's theorem find the inverse of the matrix $\begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$	CO4-App	(8)

Or

	(b)	Reduce the Q.F $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4zx$ in to a canonical form by an orthogonal transformation.	CO5-App	(16)