**A** **Reg. No. :** | | | | | | | | | | |

## Question Paper Code: 54022

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2024

Fourth Semester

Civil Engineering

15UMA422 - NUMERICAL METHODS

(Common to EEE, EIE and Chemical Engineering)

(Regulation 2015)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The sufficient condition for the convergence of iteration method is CO1- R

   (a) $|f(x)f''(x)| > [f'(x)]^2$ (b) $|\emptyset'(x)| > 1$

   (c) $|f(x)f''(x)| < [f'(x)]^2$ (d) $|\emptyset'(x)| < 1$

2. The condition for convergence of Gauss Jacobi method for solving a system of simultaneous algebraic equation is CO1- R

   (a) $|A| = 0$ (b) Orthogonal matrix

   (c) $|A| \neq 0$ (d) Diagonally dominant system

3. Newton's forward interpolation formula is mainly applied to find the value of y using a given value of x only when 'x' falls CO2- R

   (a) At the beginning of the table (b) At the middle of the table

   (c) At the end of the table (d) Far beyond the given upper value of 'x'

4. If only two pair values $(x_0, y_0)$ and $(x_1, y_1)$ are given then the Newton's forward formula reduces to CO2- R

   (a) Linear interpolation formula (b) Non-linear interpolation formula

   (c) Parabolic interpolation formula (d) Exponential polynomial

5. The process of numerical integration of a function of a single variable is called CO3- R

   (a) Trapezoidal rule (b) Simpson's rule (c) Cubature (d) Quadrature

6. The order of error in the Trapezoidal rule is CO3- R

(a) $O(h^4)$ (b) $O(h^3)$ (c) $O(h^5)$ (d) $O(h^2)$

7. Runge-Kutta method of first order is same as CO4- R

(a) Euler's method (b) Modified Euler's method

(c) Taylor series method (d) Milne's method

8. The number of prior values required to predict the next value in Milne's method is CO4- R

(a) 4 (b) 6 (c) 5 (d) 2

9. The equation $u_{xx} + u_{yy} = 0$ is of CO5- R

(a) Elliptic type (b) Parabolic type

(c) Hyperbolic type (d) Non homogeneous type

10. The interval in which the implicit formula (Crank- Nicholson) provides stable solution is CO5- R

(a) $0 < \lambda \leq 1$ (b) $0 < \lambda \leq 2$ (c) $1 < \lambda \leq 2$ (d) $0 < \lambda \leq \frac{1}{2}$

PART – B (5 x 2= 10 Marks)

11. Find the interval for a positive root of the polynomial $x^3 - 2x + 5 = 0$. CO1- App

12. Find $y$ (1) using Lagrange's interpolation formula from the given data: CO2- App

| | | | |
|---|---|---|---|
| $x$ : | 0 | 1 | 3 |
| $y$ : | 5 | 6 | 50 |

13. Find $\int_0^1 \frac{dx}{1+x}$ using two-point Gaussian quadrature formula. CO3- App

14. Find $y(1.1)$ if $y' = x + y$, $y(1) = 0$ using Taylor's series method of second order. CO4- App

15. State Crank – Nicholson difference scheme to solve a parabolic equation. CO5- R

PART – C (5 x 16= 80Marks)

16. (a) (i) Solve the following system of equations by Gauss elimination method, 2x +3y - z = 5, 4x + 4y -3z = 3, 2x – 3y + 2z = 2 CO1- App (8)

(ii) Solve the system of equations 28x+4y- z =32 , x+3y+10z =24, 2x+17y+4z = 35 by Gauss-Seidel Method. CO1- App (8)

Or

(b) (i) Find the positive root of $f(x) = 2x^3-3x-6 = 0$, by N-R method. CO1- App (8)

(ii) Determine the largest eigen value and the corresponding eigen vector of CO1- App (8)

$$A=\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ -10 & -1 & 2 \end{bmatrix}$$ by power method.

17. (a) (i) Find y at x = 43, by using Newton's forward interpolation formula from the following data, CO2- App (8)

| x | 40 | 50 | 60 | 70 | 80 | 90 |
|---|---|---|---|---|---|---|
| y | 184 | 204 | 226 | 250 | 276 | 304 |

(ii) The population of a town in the census is as given in the data. Estimate the population in the year 1996 using Newton's backward interpolation. CO2- App (8)

| Year (x) | 1961 | 1971 | 1981 | 1991 | 2001 |
|---|---|---|---|---|---|
| Population ( in 000's) | 46 | 66 | 81 | 93 | 101 |

Or

(b) (i) Using Newton's divided difference formula, find values of f (2) from the following data. CO2- App (8)

| x | 4 | 5 | 7 | 10 | 11 | 13 |
|---|---|---|---|---|---|---|
| f(x) | 48 | 100 | 294 | 900 | 1210 | 2028 |

(ii) Find f (27) by using Lagrange's formula for the data given below. CO2- App (8)

| x | 14 | 17 | 31 | 35 |
|---|---|---|---|---|
| f(x) | 68.7 | 64.0 | 44.0 | 39.1 |

18. (a) (i) Find $y'$ $and$ $y''$ at x = 1.5 from the following table, CO3- Ana (8)

| x | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
|---|---|---|---|---|---|---|
| y | 3.375 | 7.0 | 13.625 | 24.0 | 38.875 | 59 |

(ii) Find $\int_{1.6}^{2.8} f(x)\,dx$ by Simpsons $(1/3)^{rd}$ rule from the following table. CO3- Ana (8)

| x | 1.6 | 1.8 | 2.0 | 2.2 | 2.4 | 2.6 | 2.8 |
|---|---|---|---|---|---|---|---|
| f(x) | 4.95 | 6.05 | 7.39 | 9.02 | 11.02 | 13.46 | 16.44 |

Or

(b) Evaluate CO3- Ana (16)

$\int_0^1 \int_0^1 e^{x+y}\,dx\,dy$ using the Trapezoidal and Simpson's rules with

h = k = 0.5

19. (a) Solve CO4- App (16)

$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ given y(0) = 1 at x = 0.2 and x = 0.3 using

Runge – Kutta method of $4^{th}$ order.

Or

(b) (i) Find y(0.2) correct to 3 decimals given CO4- App (8)

$\frac{dy}{dx} = 1 - 2xy$, y(0) = 0 by using Taylor Series Method.

(ii) Using Milne's method find y(2) given $y' = \frac{1}{2}(x + y)$ given CO4- App (8)

y(0) = 2, y(0.5) = 2.636, y(1) = 3.595 and y(1.5) = 4.968.

20. (a) Solve CO5- App (16)

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, subject to

(i) u(0,y) = 0, $0 \le y \le 4$

(ii) u(4,y) = 12 + y, $0 \le y \le 4$

(iii) u(x,0) = 3x, $0 \le x \le 4$

(iv) u(x,4) = $x^2$, $0 \le x \le 4$ by dividing the square into 16 square meshes of side 1.

Or

(b) Using Explicit scheme solve the wave equation CO5- App (16)

$u_{tt} = u_{xx}$, $0 < x < 1$, $t > 0$, given u(x, 0) = $u_t(x,0)$ = u(0,t) =0 and

u(1,t) = 100 sin(π t). Compute u for 4 times steps with h = 0.25.