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**Question Paper Code: R2M04**

B.E./B.Tech. DEGREE EXAMINATION, APRIL 2024

Second Semester

Electronics and communication Engineering

R21UMA204- CALCULUS, COMPLEX ANALYSIS AND NUMERICAL METHODS

(Regulations R2021)

Duration: Three hours

Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The complementary function of  $(4D^2 - 3D - 1)y = 2 \sin 2x$  is \_\_\_\_\_ CO6- U  
(a)  $Ae^x + Be^{\frac{x}{4}}$  (b)  $Ae^{-x} + Be^{5x}$  (c)  $(A+Bx)e^{2x}$  (d)  $Ae^x + Be^{4x}$
2. The complete solution of  $(x^2D^2 - 3xD - 5)y = 0$  is \_\_\_\_ CO6- U  
(a)  $Ae^{-z} + Be^{5z}$  (b)  $Ae^z + Be^{5z}$  (c)  $Ae^z + Be^{-5z}$  (d)  $Ae^{-z} + Be^{-5z}$
3. If  $\vec{F}$  is a conservative field, then  $\vec{F}$  is \_\_\_\_\_ CO6- U  
(a) Solenoidal (b) Irrotational (c) 0 (d) None of these
4. If  $V$  is the volume of the region enclosed by the cube  $-1 < x, y, z < 1$  and  $\vec{F} = x^2\vec{i} + z\vec{j} + yz\vec{k}$ , then  $\iiint_V \nabla \cdot \vec{F} \, dV =$  \_\_\_\_\_ CO6- App  
(a) 3 (b) 0 (c)  $\frac{3}{2}$  (d)  $\frac{1}{3}$
5. Find the fixed points of  $f(z) = \frac{1}{z - 2i}$  CO6- U  
(a)  $i$  (b)  $2i$  (c)  $3i$  (d)  $0$
6. The function  $f(z) = \bar{z}$  is CO6- U  
(a) analytic except  $z=1$  (b) analytic every where  
(c) not analytic except  $z=0$  (d) analytic except  $z=2$

7. Simple pole is a pole of order \_\_\_\_\_ CO6- U  
 (a) 1 (b) 2 (c) 3 (d) 4
8. The poles of  $z \cot z$  is \_\_\_\_\_ CO6- U  
 (a) 0 (b)  $\pm n\pi$  (c) 1 (d)  $\pi$
9. Newton's method is also called method of \_\_\_\_\_ CO6- U  
 (a) tangents (b) slope (c) secants (d) false
10. Gauss Seidel method converges faster than \_\_\_\_\_ CO6- U  
 (a) Gauss Elimination (b) Gauss Jacobi (c) Gauss Jordan (d) Newton's

PART – B (5 x 2= 10 Marks)

11. Solve CO1 App  

$$x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = 0$$
12. Find the constant a,b,c so that  $\vec{F} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$  CO2 App  
 is irrotational.
13. Show that  $f(z) = |z|^2$  is differentiable at  $z=0$  but not analytic at  $z=0$ . CO3 App
14. Obtain the poles of  $\cot z$  CO4 App
15. Using Power method find the dominant Eigen value of  $\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$  CO5 App

PART – C (5 x 16= 80 Marks)

16. (a) (i) Solve:  $(x^2 D^2 - xD + 4)y = x^2 \sin(\log x)$  CO1- App (8)  
 (ii) Using method of variation of parameters solve CO1- App (8)  
 $(D^2 + a^2)y = \tan ax$
- Or
- (b) (i) In a culture of bacteria the rate of increase is proportional to CO1- App (8)  
 the number present. If it is found that the number doubles in 4  
 hours, how many may be expected at the end of 12 hours?
- (ii) Solve:  $[(x+1)^2 D^2 + (x+1)D + 1]y = 2 \cos[\log(x+1)]$  CO1- App (8)
17. (a) Verify Divergence theorem for  $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - xz)\vec{j} + (z^2 - xy)\vec{k}$  CO2- App (16)  
 over the rectangular parallelepiped  $x = 0, x = a, y = 0, y = b, z = 0, z = c$ .

Or

- (b) (i) Prove that  $\vec{F} = (x^2 + xy^2)\vec{i} + (y^2 + x^2y)\vec{j}$  is irrotational vector and find the Scalar potential such that  $\vec{F} = \nabla\phi$ . CO2- App (8)
- (ii) Using Green's theorem, find the value for  $\int_C (3x^2 - 8y^2) dx + (4y - 6xy)dy$  where C is the boundary of the region defined by  $X = 0, Y = 0, X + Y = 1$  in the XY plane. CO2- App (8)
18. (a) (i) Using Milne Thomson method, find the Analytic function given that  $u + v = \frac{\sin 2x}{\cosh 2y - \cos 2x}$  CO3- App (8)
- (ii) Find the image of  $|z - 3i| = 3$  under the transformation  $w = \frac{1}{z}$  CO3- App (8)
- Or
- (b) (i) Using Milne Thomson method, find the Analytic function given that  $u = \frac{\sin 2x}{\cosh 2y - \cos 2x}$  CO3- App (8)
- (ii) Show that  $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4 \frac{\partial^2}{\partial z \partial \bar{z}}$  CO3- App (8)
19. (a) (i) Expand  $\frac{z-1}{(z+2)(z+3)}$  as Laurent's series valid in the region  $2 < |z| < 3$  CO4- App (8)
- (ii) Evaluate using Cauchy's Residue theorem for  $f(z) = \int_C \frac{3z^2 + z - 1}{(z^2 - 1)(z - 3)} dz$ , where 'C' is the circle  $|z| = 2$ . CO4- App (8)
- Or
- (b) Using contour integration, Evaluate  $\int_0^{2\pi} \frac{1}{5 + 4 \sin \theta} d\theta$  CO4- App (16)
20. (a) (i) Solve the equation  $e^x - 3x = 0$  by iteration method CO5- App (8)
- (ii) Solve  $28x + 4y - z = 32; x + 3y + 10z = 24; 2x + 17y + 4z = 35$  by Gauss - Seidel method CO5- App (8)
- Or

(b) (i) Using Power method find numerically largest Eigen value of CO5- App (8)

$$\begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$$

(ii) Solve  $4x + 2y + z = 14$ ,  $x + 5y - z = 10$ ,  $x + y + 8z = 20$  by CO5- App (8)  
Gauss Jordan method