A   Reg. No. :

Question Paper Code: U2M04

B.E./B.Tech. DEGREE EXAMINATION, MAY 2024

Second Semester

Electronics and communication Engineering

21UMA204- Calculus,Complex analysis and Numerical methods

(Regulations 2021)

Duration: Three hours   Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. $\frac{1}{(D-m)^2} e^{mx}$ = _____   CO6-U

   (a) xe^{mx} (b) $x^2 e^{mx}$ (c) $\frac{x^2}{2} e^{mx}$ (d) $\frac{x^2}{m} e^{mx}$

2. The complementary function of $(4D^2 - 3D-1)y = 2 \sin 2x$ is _____   CO6-U

   (a) $Ae^x + Be^{-\frac{x}{4}}$ (b) $Ae^{-x} + Be^{5x}$ (c) $(A+Bx)e^{2x}$ (d) $Ae^x + Be^{4x}$

3. $\text{Div } \bar{r}$ = _____   CO2-App

   (a) 0   (b) 1   (c) 3   (d) $\bar{r}$

4. Divergence of vector $x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ at (1, 2, -3) is _____   CO2-App

   (a) 8   (b) 4   (c) -3   (d) 0

5. The critical point of the transformation $w = z + \frac{1}{z}$ are _____   CO3- App

   a) ± 1 b) ± 2 c) $\pm i$ d) $-i$

6. The function $f(z) = \frac{1}{z^2+4}$ is not analytic at $z =$ _____   .   CO3- App

   (a) 2   b) -2   c) 2i   d) $\pm 2i$

7. If f(z) is analytic at all points inside and on a simple closed curve c,   CO6-U

then $\int\limits_C f(z)dz$ = ------

(a) $2\pi i$ (b) -$2\pi i$ (c) $4\pi i$ (d) 0

8. Simple pole is a pole of order _____   CO6-R

(a) 1   (d) 2   (c) 3   (d) 4

9. Iteration method converges if $|g^1(x)|$ _____   CO6-R

(a) >1   (b)<1   (c)=0   (d) >0

10. Order of convergence of iteration method is   CO5-U

(a) 1   (d) 2   (c) 3   (d) 0

PART – B (5 x 2= 10Marks)

11. Find Particular integral for $(D^2 - 2D + 1)y = \cosh x$.   CO1-App

12. Find $\nabla\varphi$, if $\varphi = x^2 + y^2 + z^2$ at (1, -1, 1).   CO2-App

13. Find the fixed point of $w = \dfrac{2z-5}{z+4}$ CO3-App

14. Evaluate $\int\limits_c \dfrac{z}{z-2}dz$ where C is $|z| = 2$ CO4-App

15. State Newton's Iterative formula   CO5-R

PART – C (5 x 16= 80Marks)

16. (a) (i) Using method of variation of parameters solve $(D^2 + a^2)y = \sec ax$ CO1-App   (8)

(ii) At the start of an experiment, there are 100 bacteria. If the bacteria follow an exponential growth pattern with rate k = 0.02. What will be the population after 5 hours? How long will it take for the population to double?   CO1- App   (8)

Or

(b) (i) Solve: $(x^2D^2 - xD + 4)y = x^2 \sin(\log x)$ CO1- App   (8)

(ii) Solve: $(D^2 - 4D + 3)y = \cos 2x + e^{2x}$ CO1- App   (8)

17. (a) Verify Gauss Divergence theorem for $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ where S is the surface of the cuboid formed by the planes $x = 0, x = a,$ $y = 0, y = b, z = 0 \,\&\, z = c.$ CO2-App   (16)

U2M04

Or

(b) Verify Green's theorem for $\int_C \mathbf{x^2 dx} + \mathbf{xy dy}$, whereC is bounded by   CO2 -App   (16)

x = 0,
x = a, y = 0, y = a

18. (a) (i) Find the image of $|z - 3i| = 3$ under the transformation $w = \dfrac{1}{z}$ CO3-App   (8)

(ii) If f(z) = u +iv is a regular function of z in a domain D the   CO3-App   (8)
following relation hold in D. $\nabla^2 |f(z)|^2 = 4|f'(z)|^2$.

Or

(b) (i) Find the bilinear transformation from -i,0,i to -1,i,1.   CO3-App   (8)

(ii) Find the analytic functions f(z) = u +iv given that   CO3-App   (8)
$2u + v = e^x(\cos y - \sin y)$

19. (a) (i) Evaluate using Cauchy's Residue theorem for   CO4-App   (8)

$f(z) = \int_C \dfrac{3z^2 + z - 1}{(z^2 - 1)(z - 3)}\, dz$, where 'C' is the circle $|z| = 2$.

(ii) Evaluate $f(z) = \dfrac{1}{(z+1)(z+3)}$ in Laurent series valid for the   CO4-App   (8)
region
$1 < |z| < 3$.

Or

(b) Using contour integration, to find the value of $\int\limits_{0}^{2\pi} \dfrac{d\theta}{13 - 5\cos\theta}$ CO4-App   (16)

20. (a) (i) Solve for a positive root of $3x - \cos x - 1 = 0$ by Newton's   CO5-App   (8)
Raphson method .
(ii) Solve 4x + 2y +z = 14, x + 5y -z = 10, x + y + 8z =20 by   CO5- App   (8)
Gauss Elimination method

Or

(b) (i) Using Power method find numerically largest Eigen value of   CO5- App   (8)
$\begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$

(ii) Solve by using convolution theorem Solve 28x+4y-z = 32;   CO5- App   (8)
x+3y+10z = 24 ; 2x+17y+4z = 35 by Gauss - Seidel method

U2M04