A **Reg. No. :** | | | | | | | | | | | | |

Question Paper Code: R2M03

B.E./B.Tech. DEGREE EXAMINATION, MAY 2024

Second Semester

Computer Science and Engineering

R21UMA203- DIFFERENTIAL EQUATIONS AND COMPLEX ANALYSIS

(Regulations R2021)

(Common to IT, Cyber Security & IOT Engineering Branches)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. Particular Integral of $(D^2)y = 6x$ CO1- App

(a) $3x^3$ (b) $-3x^3$ (c) x^3 (d) $\frac{1}{x^3}$

2. Particular Integral of $(D^2 + 5)y = \sin 2x$ CO1- App

(a) $\sin 2x$ (b) $-\sin 2x$ (c) $\cos 2x$ (d) $-\frac{x}{4}\cos 2x$

3. The critical point of the transformation $w = 2z + \frac{1}{z}$ are _____ CO3- App

a) ± 1 b) ± 2 c) $\pm i$ d) $-i$

4. The transformation w=1/z is known as _______ CO3- U

(a) Rotation b) reflection c) translation d) inversion

5. Greens theorem is a relation between ______ CO2- U

(a) two volume integrals (b) line integral and surface integral

(c) surface integral and volume integral (d) volume integral and line integral

6. The real and imaginary parts of an analytic function are _____ CO3- U

(a) Harmonic (b) orthogonal

(c) satisfies Laplace equation (d) All the above are true

7. $\int_C \frac{e^z}{z-2} dz$ where C is the unit circle with center as origin is _______ CO4- App

(a) 0 (d) 1 (c) 2 (d) π

8. The poles of $f(z) = \frac{z^2+1}{1-z^2}$ are CO6- U

(a)1,0 (b)0,0 (c)1,2 (d)-1,1

9. The type of $3\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial y} + 5\frac{\partial^2 u}{\partial y^2} = x$ CO5- App

(a) parabolic (b) hyperbolic (c) elliptic (d) cyclic

10. In a one dimensional wave equation, c^2= ___. CO5- U

(a) T^2/m^2 (b) T/m (c) T/m^2 (d) T^2/m

PART – B (5 x 2= 10 Marks)

11. Compute the Particular Integral of $(D^3 - 3D)y = 4e^{-x}$ CO1 App

12. Show that $\vec{F} = z\vec{i} + x\vec{j} + y\vec{k}$ is Solenoidal CO2 App

13. Prove that $u = e^x \cos y$ is harmonic function CO3 App

14. Using Cauchy's integral formula , Evaluate $\int_c \frac{e^{-z}}{z+1} dz$ where C is $|z| = 5$ using Cauchy integral formula CO4 App

15. Find the particular integral of $\left(D^2 + 3DD' + 5D'^2\right)Z = e^{x+2y}$ CO5 App

PART – C (5 x 16= 80Marks)

16. (a) (i) Solve CO1 App (8)

$\left(D^2 + 3D + 2\right)y = e^{-2x} + \cos 2x$

(ii) Using method of variation of parameters solve CO1 App (8)
$(D^2 + a^2)y = \text{cosec } ax$

Or

(b) (i) Solve the differential equation $(x^2D^2 - 3xD - 5)y = x^2 \sin(\log x)$ CO1 App (8)

(ii) Solve the differential equation CO1 App (8)

$[(x+1)^2 D^2 + (x+1)D + 4]y = \sin[\log(x+1)]$

17. (a) Verify Gauss divergence theorem for the vector function $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ over the cube bounded by the planes x = 0, x = a, y = 0, y = b, z = 0, z = c — CO2 App (16)

Or

(b) Verify Stokes theorem for a vector field defined by $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ in the rectangular region in the XOY plane bounded by the lines $x = \pm a, y = 0, and\ y = b.$ — CO2 App (16)

18. (a) (i) Determine the analytic function whose real part is $(x - y)(x^2 + 4xy + y^2)$ — CO3 App (8)

(ii) Determine the bilinear transformation which maps $z = 0, 1, \infty$ onto $w = -5, -1, 3$ — CO3 App (8)

Or

(b) (i) If f(z)=u+iv is an analytic function then Prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f^1(z)|^2$ — CO3 App (8)

(ii) Determine the image of $|z - 3i| = 3$ under the transformation $w = \frac{1}{z}$ — CO3 App (8)

19. (a) (i) Using Contour integration — CO4 App (8)

Evaluate $\int_0^\infty \frac{x^2 dx}{(x^2 + 9)(x^2 + 25)}$

(ii) Cauchy's Residue theorem, Evaluate $\int_c \frac{2z + 7}{(z - 3)(z - 1)(z + 2)} dz$ where C is the circle $|z| = 4$ — CO4 App (8)

Or

(b) (i) Evaluate f(z) = $\frac{\mathbf{7z - 2}}{\mathbf{z(z + 1)(z - 2)}}$ in Laurent's series valid in the region $\mathbf{1 < |z + 1| < 3}$ — CO4 App (8)

(ii) Using Contour integration, evaluate $\int_0^{2\pi} \frac{1}{13 - 5\cos\theta} d\theta$ — CO4 App (8)

20. (a) A tightly String with fixed end points x=0 and x=l is initially at rest in its equilibrium position. If its set vibrating giving each point at velocity $\lambda(lx-x^2)$. Determine the displacement function y(x,t). CO5 App (16)

Or

(b) (i) Solve : CO5 App (8)

$$(D^2 - 4DD' + 4D'^2)Z = e^{2x+y} + xy$$

(ii) Solve : CO5 App (8)

$$(3z - 4y)p + (4x - 5z)q = 5y - 3x$$