

A

Reg. No. :

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

**Question Paper Code: R2M10**

B.E./B.Tech. DEGREE EXAMINATION, MAY 2024

Second Semester

Computer Science and Design

R21UMA210- DIFFERENTIAL EQUATIONS, FOURIER SERIES & TRANSFORMS

(Regulations R2021)

(Common to Artificial Intelligence and Data Science)

Duration: Three hours

Maximum: 100 Marks

Answer All Questions

PART A - (10 x 1 = 10 Marks)

- Particular integral of  $y'' + 4y' + 4y = 0$  is \_\_\_\_\_.  
(a)  $xe^{-2x}$  (b)  $xe^{2x}$  (c) 0 (d)  $e^{2x}$  CO1-App
- The solution of  $(D^3 + D^2 - D - 1)y = 0$  is \_\_\_\_\_.  
(a)  $Ae^x + Bxe^x + Cx^2e^x$  (b)  $(Ax + B)e^x + Ce^{-x}$   
(c)  $e^{-x} + (\cos 2x + i \sin 2x)$  (d)  $(Ax + B)e^{-x} + Ce^x$  CO1-App
- $L(\cosh at) =$  \_\_\_\_\_.  
(a)  $\frac{s}{s^2 - a^2}$  (b)  $\frac{a}{s^2 - a^2}$  (c)  $\frac{s}{s^2 + a^2}$  (d)  $\frac{a}{s^2 + a^2}$  CO2- App
- $L[f(at)] =$  \_\_\_\_\_.  
(a)  $\frac{s}{a} F\left(\frac{s}{a}\right)$  (b)  $a F\left(\frac{s}{a}\right)$  (c)  $\frac{1}{a} F\left(\frac{s}{a}\right)$  (d)  $a F(s)$  CO2-App
- If  $Z[f(t)] = F(z)$ , then  $f(0) =$  \_\_\_\_\_.  
(a)  $\lim_{z \rightarrow \infty} F(z)$  (b)  $\lim_{z \rightarrow 0} F(z)$  (c) 0 (d) 1 CO6 - U
- $Z[u(n-2)] =$  \_\_\_\_\_.  
(a)  $\frac{1}{z(z-1)}$  (b)  $\frac{1}{z(z+1)}$  (c)  $\frac{z}{z(z-1)}$  (d)  $\frac{1}{(z-1)}$  CO3 - App
- If  $f(-x) = -f(x)$ , then  $f(x)$  is said to be an \_\_\_\_\_.  
(a) Odd Function (b) Even Function (c) Periodic function (d) Self Reciprocal CO6 - U

8. The root mean square value of  $f(x) = x^2$  in  $(0, 1)$  is ----- CO4 – App
- (a) 1 (b)  $1/2$  (c)  $1/\sqrt{3}$  (d) 2
9. In Modulation property,  $F[f(x) \cos ax] =$  \_\_\_\_\_ CO5 - App
- (a)  $\frac{1}{2}[F(s+a) - F(s-a)]$  (b)  $\frac{1}{2}[F(s+a) + F(s-a)]$
- (c)  $[F(s+a) - F(s-a)]$  (d)  $F(s+a) + F(s-a)$
10.  $F_s[e^{-ax}] =$  \_\_\_\_\_ CO5 – App
- (a)  $\sqrt{\frac{2}{\pi}} \frac{s}{s^2+a^2}$  (b)  $\sqrt{\frac{2}{\pi}} \frac{a}{s^2+a^2}$  (c)  $\sqrt{\frac{2}{\pi}} \frac{a^2}{s^2+a^2}$  (d)  $\sqrt{\frac{2}{\pi}} \frac{s^2}{s^2+a^2}$

PART – B (5 x 2= 10Marks)

11. Find Particular integral for CO1 -App
- $(D^2 - 2D + 1)y = \cosh x$
12. Evaluate  $L[\cos at]$  CO2-App
13. Find  $Z[4 \cdot 2^n + 3(-1)^n]$  CO3 -App
14. Give the expression for the Fourier series coefficient  $b_n$  for the function CO4- App
- $f(x) = x^2$  defined in  $-2 \leq x \leq 2$ .
15. Find the Fourier Sine Transform of  $f(x) = e^{-x}, x > 0$  CO5- App

PART – C (5 x 16= 80Marks)

16. (a) (i) Solve  $(D^2 - 3D + 2)y = 2e^x + 2\cos 2x$  CO1- App (8)
- (ii) Solve  $(3z - 4y)p + (4x - 2z)q = 2y - 3x$  CO1-App (8)
- Or
- (b) (i) Solve  $(x^2D^2 + 4xD + 2)y = x + \frac{1}{x}$  CO1-App (8)
- (ii) Solve  $(D^2 - D - 6)y = 3e^{4x} + 5$  CO1-App (8)
17. (a) (i) Find the Laplace transform of CO2-App (8)
- $$f(t) = \begin{cases} \sin \omega t, & 0 < t < \frac{\pi}{\omega} \\ 0, & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases} \quad \text{and} \quad f\left(t + \frac{2\pi}{\omega}\right) = f(t)$$
- (ii) Solve by using convolution theorem CO2-App (8)
- $$L^{-1}\left[\frac{s^2}{(s^2 + 2^2)(s^2 + 3^2)}\right]$$

Or

- (b) (i) Solve the differential equation  $\frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} - 3y = \sin t$  if  $y(0) = y'(0) = 0$  by using Laplace transform method. CO2-App (8)
- (ii) Find the Laplace Transforms of  $te^{-2t} \sin t$  CO2-App (8)

18. (a) (i) Evaluate CO3 -App (8)
- (i)  $Z\left(\cos \frac{n\pi}{2}\right)$  (ii)  $Z\left(\sin \frac{n\pi}{2}\right)$

- (ii) Using convolution theorem find CO3 -App (8)

$$Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right]$$

Or

- (b) (i) Solve the difference equation CO3- App (8)
- $y_{n+2} - 6y_{n+1} + 8y_n = 5^n$  given that  $y_0 = 0, y_1 = 0$
- (ii) Evaluate CO3 -App (8)

$$Z^{-1}\left[\frac{z}{z^2 + 5z + 6}\right]$$

19. (a) Find the Fourier series of  $f(x) = x^2$  in  $0 < x < 2\pi$  CO4-App (16)
- Or

- (b) (i) The table of values of the function  $y = f(x)$  is given below: CO4-App (10)

x:	0	T/6	T/3	T/2	2T/3	5T/6	T
y:	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Find a Fourier series up to the third harmonic to represent  $f(x)$  in terms of  $x$ .

- (ii) Find the half range sine series for  $f(x) = x$  in  $(0, \pi)$  CO4-App (6)

20. (a) Show that the Fourier transform of

CO5-App (16)

$$f(x) = \begin{cases} a^2 - x^2 & |x| < a \\ 0 & |x| > a > 0 \end{cases} \text{ is } 2\sqrt{\frac{2}{\pi}} \left[ \frac{\sin sa - sa \cos sa}{s^3} \right]$$

Hence deduce  $\int_0^{\infty} \frac{\sin t - t \cos t}{t^3} dt = \frac{\pi}{4}$  and

$$\int_0^{\infty} \left( \frac{\sin t - t \cos t}{t^3} \right)^2 dt = \frac{\pi}{15}$$

Or

(b) (i) Find Fourier Cosine and Sine Transform of  $e^{-ax}$  hence

CO5-App (8)

evaluate  $\int_0^{\infty} \frac{dx}{(x^2 + 9)^2}$

(ii) Evaluate

CO5-App (8)

$$\int_0^{\infty} \frac{x^2 dx}{(x^2 + a^2)(x^2 + b^2)}$$