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Question Paper Code: U2M10

B.E./B.Tech. DEGREE EXAMINATION, MAY 2024

Second Semester

Computer Science and Design

21UMA210- Differential equations, Fourier series & Transforms

(Regulations 2021)

(Common to Artificial Intelligence and Data Science)

Duration: Three hours

Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

- The auxiliary equation of the equation $x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = e^x$ is _____. CO1-App
(a) $m^2 - 4m + 5 = 0$ (b) $m^2 + 3m - 2 = 0$ (c) $m^2 + 3m + 2 = 0$ (d) $2m^2 + 5m - 7 = 0$
- $\frac{1}{D^2}(\cos x) =$ _____. CO6-R
(a) $\sin x$ (b) $-\cos x$ (c) $\cos x$ (d) $\tan x$
- $L(\sin h at) =$ _____. CO2-App
(a) $\frac{s}{s^2 - a^2}$ (b) $\frac{a}{s^2 - a^2}$ (c) $\frac{s}{s^2 + a^2}$ (d) $\frac{a}{s^2 + a^2}$
- $L(te^{-3t})$ CO2-App
(a) $-z \frac{d}{dz} F(Z)$ (b) $z \frac{d}{dz} F(Z)$ (c) $\frac{d}{dz} F(Z)$ (d) $z^n \frac{d}{dz} F(Z)$
- $Z[n f(n)] =$ _____. CO3- App
(a) $-z \frac{d}{dz} F(Z)$ (b) $z \frac{d}{dz} F(Z)$ (c) $\frac{d}{dz} F(Z)$ (d) $z^n \frac{d}{dz} F(Z)$
- The Z transform of $n2^n$ is _____. CO3- App
(a) $\frac{2z}{(z-2)^2}$ (b) $\frac{z}{(z-2)^2}$ (c) $\frac{2z}{(z+2)^2}$ (d) $\frac{z}{(z+2)^2}$

7. The root mean square value of $f(x) = x^2$ in $(0, 1)$ is -----.
- (a) 1 (b) $1/2$ (c) $1/\sqrt{3}$ (d) 2 CO4-App
8. The constant term in the Fourier expansion for $f(x) = k$, $(0, 2\pi)$ is CO4-App
- (a) $2k$ (b) $3k$ (c) k (d) 0
9. Convolution theorem on Fourier Transform is $F[f(x)*g(x)] =$ CO6-U
- _____
- (a) $F(s).G(s)$ (b) $f(s).g(s)$ (c) $F(s)*G(s)$ (d) $f(s)*g(s)$
10. If $f(x)$ is an even function then $\int_{-a}^a f(x)dx =$ CO6-U
- (a) 0 (b) $\frac{1}{2} \int_0^a f(x)dx$ (c) $2 \int_0^a f(x)dx$ (d) $\int_0^a f(x)dx$

PART – B (5 x 2= 10Marks)

11. Find Particular integral for CO1-App
- $(D^2 - 2D + 1)y = \cosh x$
12. Evaluate $L[\cos at]$ CO2-App
13. Evaluate $Z\left(\frac{1}{n+1}\right)$ CO3-App
14. Give the expression for the Fourier series coefficient b_n for the function $f(x) =$ CO4-App
- x^2 defined in $-2 \leq x \leq 2$.
15. Find the Fourier Sine Transform of $f(x) = e^{-x}, x > 0$ CO5-App

PART – C (5 x 16= 80Marks)

16. (a) (i) Solve $(D^2 - 3D + 2)y = 2e^x + 2\cos 2x$ CO1-App (8)
- (ii) Solve $(D^2 - D - 6)y = 3e^{4x} + 5$ CO1- App (8)
- Or
- (b) (i) Solve $(D^2 - 3D + 2)y = 2e^x + 2\cos 2x$ CO1- App (8)
- (ii) Solve $(3z - 4y)p + (4x - 2z)q = 2y - 3x$ CO1- App (8)
17. (a) $f(t) = \begin{cases} t, & 0 < t < a \\ 2a - t, & a < t < 2a \end{cases}$ and CO2-App (16)
- Find the Laplace transform of $f(t + 2a) = f(t)$

Or

- (b) Solve by the convolution theorem CO2 -App (16)

$$L^{-1}\left[\frac{s}{(s^2 + a^2)^2}\right]$$

18. (a) Using convolution theorem find the inverse Z-transform CO3-App (16)

$$Z^{-1}\left(\frac{8z^2}{(2z-1)(4z-1)}\right)$$

Or

- (b) (i) Evaluate $Z[a^n \cos n\theta]$ and $Z[a^n \sin n\theta]$ CO3-App (8)

(ii) Evaluate CO3-App (8)

$$Z^{-1}\left[\frac{z}{z^2 + 4z + 3}\right]$$

19. (a) Find the Fourier series of $f(x) = x^2$ in $(-\pi, \pi)$ of periodicity 2π . CO4-App (16)

Or

- (b) (i) The table of values of the function $y = f(x)$ is given below: CO4-App (8)

x:	0	1	2	3	4	5
Y:	4	8	15	7	6	2

Find a Fourier series up to the third harmonic to represent $f(x)$ in terms of x .

- (ii) Find the half range sine series for $f(x) = x^2$ in $(0, l)$ CO4-App (8)

20. (a) Find the Fourier transform of $f(x) = \begin{cases} 1-|x|, & \text{if } |x| \leq 1 \\ 0 & \text{if } |x| > 1 \end{cases}$ and CO5-App (16)

hence deduce that i) $\int_0^{\infty} \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$ ii) $\int_0^{\infty} \left(\frac{\sin t}{t}\right)^4 dt = \frac{\pi}{3}$

Or

- (b) (i) Find the Fourier sine & cosine transform of e^{-ax} CO5-App (8)

- (ii) Evaluate $\int_0^{\infty} \frac{dx}{(x^2 + 25)^2}$ CO5-App (8)

