A   **Reg. No. :** ☐☐☐☐☐☐☐☐☐☐☐☐

Question Paper Code: U4M26

B.E./B.Tech. DEGREE EXAMINATION, APRIL 2024

Fourth Semester

Computer Science and Design

21UMA426- PROBABILITY AND STATISTICAL TECHNIQUES

(Common to Artificial Intelligence and Data Science Engineering)

(Regulations 2021)

Duration: Three hours   Maximum: 100 Marks

Answer All Questions

PART A - (10 x 1 = 10 Marks)

1. Probability of an impossible event is   CO6- U

    (a) 1   (b)10   (c)0   (d) 100

2. The mean of the random variable is denoted by   CO6- U

    (a) $E(X)$ (b) $E(X^2)$ (c) 0   (d) 1

3. Which of the following distribution has equal mean and Variance?   CO6 -U

    a) Geometric   (b)Poisson   (c) Normal   (d) Binomial

4. For Binomial Distribution   CO6 -U

    (a) $E(X)=\sigma^2$ (b) $E(X)>\sigma^2$ (c) $E(X)<\sigma^2$ (d) $E(X)\geq\sigma^2$

5. For a set of five bivariate data(x, y) covariance is 10. Variance of x and y   CO3 – App
    are 25 and 16 respectively. The Karl Pearson coefficient of correlation is

    (a) 1   (b) -1   (c) 1/2   (d) 1/4

6. The joint probability density function is $f(x, y) = k$, $0 < x < 2$, $0 < y < 1$.   CO3 – App
    Estimate K =

    a)4   b)-1/2   c) 1/2   d)5

7. Estimate and estimator are: CO 6 – U

    (a) Same   (b)Different   (c)Maximum   (d) Minimum

8. If T is an unbiased estimator for θ, then T^2 is a ………….. Estimator for θ^2.   CO 6 – U

   (a) unbiased   (b) biased   (c) Both (a) & (b)   (d) None of the above

9. Large sample size is   CO6- U

   (a) 30   (b) >30   (c) <30   (d) none of these

10. F-test is used to test for equality of _____   CO6- U

   (a) Mean   (b) Variance   (c) ratio   (d) all the above

<div align="center">PART – B (5 x 2= 10Marks)</div>

11. In the probability density function $f(x) = k(8 - x), 0 \le x \le 2$, Find K   CO1 -App

12. For Binomial distribution mean is 4 and variance is 2, Find $P(X = 7)$ CO2- App

13. If Correlation coefficient $\gamma = 0.4, \sigma_x = 5, \sigma_y = 2$, find the covariance value   CO3 -App

14. Define: point estimator.   CO6 -U

15. Define: Type I Error &Type II Error.   CO6 -U

<div align="center">PART – C (5 x 16= 80Marks)</div>

16. (a)   (i) A RV X has the following distribution   CO1 – App   (8)

x	0	1	2	3	4	5
P(X	a	3a	5a	7a	9a	10a

Compute Mean and Variance 8

(ii) The joint probability mass function of (X,Y) is given by   CO1 – App   (8) $P(x, y) = k(x + 3y)$ $x = 0, 1, 4$ $y = 1, 2, 3$ Compute marginal distribution function , and conditional distribution

<div align="center">Or</div>

(b) (i) The cumulative distribution function of a random variable X is   CO1 – App   (8) $F(x) = 1 - (1 + x)e^{-x}, x > 0$. Find the probability density function of X, mean and variance.

(ii) If X and Y are two random variables with joint pdf   CO1 – App   (8) $f(x, y) = K(6 - x - y), 0 < x < 2, 2 < y < 4$

Compute (a) K (b) Marginal density function of x (c) E (X)

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17. (a) (i) Compute the moment generating function of Poisson CO2 –App (8)
distribution and hence Compute it's mean and variance.

(ii) If X is a normal variate with mean 30 and variance 25. CO2 –App (8)
Compute the probabilities that Compute

$P(25 \leq X \leq 40)$, $P(X \geq 36)$, $P(|X - 34| > 3)$

Or

(b) (i) A random variable X has a uniform distribution over (-3,3) CO2 –App (8)
compute

(i) P(X < 2) (ii) P(|X| < 1) and (iii) P(X > - 1)

(ii) Compute the moment generating function of Exponential CO2 –App (8)
distribution and hence Compute it's mean and variance

18. (a) (i) Obtain the rank Correlation coefficient for the following data: CO3 App (8)

X	12	15	17	18	12	16	15	27
Y	14	10	14	13	16	10	14	15

(ii) Obtain the Correlation coefficient for the following heights (in CO3- App (8)
inches) of fathers X and their sons Y.

X	58	56	59	57	58	50	60	64
Y	67	68	65	68	72	72	69	75

Or

(b) (i) Joint pdf of X and Y is CO3 -App (8)

$$f(x, y) = \begin{cases} x + y & ,0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & elsewhere \end{cases}$$

Compute Regression Equations

(ii) If $\sigma_1 = 2$, $\sigma_2 = \sigma_3 = 3$, $r_{12} = 0.7$, $r_{23} = r_{31} = 0.6$ Compute CO3- App (8)

(i)$r_{23.1}$ (ii) $R_{1.23}$ (iii) $b_{13.2}$ (iv) $b_{12.3}$

19. (a) (i) If $x_1, x_2, x_3 \cdots x_n$ is a random sample from a normal population CO4 - App (8)

N(μ, 1). Show that $t = \dfrac{1}{n} \displaystyle\sum_{i=1}^{n} x_i^2$ is an unbiased estimator of

$\mu^2 + 1$.

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(ii) The mean weekly sales of soap bars in departmental stores were 146.3 bars per store. After an advertising campaign the mean weekly sales in 400 stores for a typical week increased to 153.7 and showed a standard deviation of 17.2. Was the advertising campaign successful?   CO4- App   (8)

Or

(b) A sample of size n is drawn from each of the four normal populations which have the same variance σ^2. The means of the four populations are a + b + c, a + b − c, a − b + c and a − b − c. What are the MLE's for a, b, c and σ^2.   CO4 - App   (8)

(i) Show that $\dfrac{\sum x_i \left[\sum x_i - 1\right]}{n\,(n-1)}$ is an unbiased estimate of θ^2, for the sample $x_1, x_2, x_3 \cdots x_n$ drawn on X which takes the values 1 or 0 with respective probabilities θ and $(1 - \theta)$.   CO4 – App   (8)

20. (a) (i) A certain injection administered to each of 12 patients resulted in the following increases of blood pressure: 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4 can it be classified that the injection will be, in general, accompanied by an increase in BP?   CO5 – Ana   (8)

(ii) The table gives the number of aircraft accidents that occurred during the various days of the week. Test whether the accidents are uniformly distributed over the week.   CO5 – Ana   (8)

Days	Mon	Tue	Wed	Thu	Fri
No. of accidents	8	12	9	14	17

Or

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(b) (i) Two researchers adopted different sampling techniques while CO5 – Ana (8)
investigating the same group of students to find the number of
students falling into different intelligence level. The results are as
follows

Research	Below Average	Average	Above Average	Excellent
X	40	33	25	2
Y	86	60	44	10

(ii) Two independent samples of sizes 9 and 7 from a normal CO5 – Ana (8)
population had the following values of the variables.

Sample I	17	13	12	15	12	14	16	14	15
Sample II	16	19	13	16	18	13	15	--	--

 Do the estimates of the population variance differ significally at
5% level?

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