A **Reg. No. :** | | | | | | | | | | | | |

Question Paper Code: U2M08

B.E./B.Tech. DEGREE EXAMINATION, MAY 2024

Second Semester

Computer Science and Business Systems

21UMA208- LINEAR ALGEBRA AND NUMERICAL METHODS

(Regulations 2021)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. If the Eigen value of a matrix A are 1,2,3 then the Eigen value of A^T CO1-App

   (a)2,4,6 (b) 1,4,9 (c) 2,8,18 (d)1,2,3

2. If the Eigen value of the matrix $A=\begin{pmatrix}1 & 1\\ 3 & -1\end{pmatrix}$ are 2,-2 then the Eigen value of A^{-1} are___ CO1-App

   (a) $\frac{1}{2},-\frac{1}{2}$ (b) 2,-2 (c) 1,-1 (d) 1,3

3. Solve the linear system 5x+4y=15,3x+7y=12 gauss –Jordan method CO2-App

   (a) $\frac{57}{23},\frac{15}{23}$ (b) $\frac{15}{23},\frac{15}{23}$ (c) $\frac{5}{23},\frac{15}{23}$ (d) $\frac{57}{23},\frac{5}{23}$

4. By Gauss elimination method, solve $x+y=2$, $2x+3y=5$ CO2-App

   (a)1,2 (b) 1,1 (c) 1,0 (d) 0,1

5. Gauss Seidel method iteration converges if the coefficient matrix is ________dominant CO3- U

   (a) Squarely (b) Logically (c) Diagonally (d) Symmetrically

6. The order of convergence of Newton's method is ________ CO3- U

   (a) 1 (b) 2 (c) 3 (d) 0

7. In a vector space V, for every $x, y \in V$ then property x+y=y+x is known as________ CO6-R

(a) Commutative (b) Associative (c) identity (d) Inverse

8. The dim(R^3) is _______ CO6-U

(a) 1 (b) 2 (c) 3 (d) 0

9. $\langle x, x\rangle = 0$ if and only if then find x _______ CO6-U

(a) x=1 (b) $x \neq 1$ (c) x=0 (d) $x \neq 0$

10. The norm of $(3,-4,0)$ is _________ CO6-R

(a) 3 (b) -4 (c) 0 (d) 5

PART – B (5 x 2= 10 Marks)

11. If the Eigen value of the matrix $A = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$ are 2,-2 then find the Eigen value of A^{-1} CO1-App

12. Apply Gauss –Jordan method solve the linear system $x + y = 2; 2x + 3y = 5$. CO2-App

13. Explain Newton's backward interpolation formula CO6-R

14. Find the matrix of $T: V_2(R) \to V_3(R)$ given by $T(a,b) = (a+3b, 0, 2a-4b)$ for the standard Basis of $V_2(R)$ CO4-App

15. Explain rank-nullity theorem CO6-U

PART – C (5 x 16= 80Marks)

16. (a) Using Cayley Hamilton theorem find A^4 and A^{-1} when A= CO1-App (16)

$$\begin{pmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix}.$$

Or

(b) Apply the orthogonal transformation reduce the following quadratic forms into canonical form $Q = 6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4zx$, find its rank, index, signature and nature CO1- App (16)

17. (a) (i) Apply Gauss elimination method to solve 2x+y+4z=12,8x-3y+2z=1,4x+11y-z=33 CO2-App (8)

(ii) Apply Gauss Jordan method to solve 10x+y+z=12, 2x+10y+z=13, x+y+5z=7 CO2-App (8)

Or

(b) Solve by using LU decomposition method CO2 -App (16)
2x+5y+2z=18;x+2y+3z=14;3x+y+5z=20

18. (a) (i) Using Lagrange's interpolation formula calculate the profit in the year 2000 from the following data : CO3-App (8)

year	1997	1999	2001	2002
Profit (Rs.in lakhs)	43	65	159	248

(ii) Apply Newton Raphson Method Calculate a root of $x\log_{10} x - 1.2 = 0$ correct to 3 decimals. CO3-App (8)

Or

(b) Calculate the dominant Eigen value and corresponding Eigen vector of A. if CO3-App (16)

$$A=\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

19. (a) Construct the linear transformation $T: V_3(R) \to V_3(R)$ determine by the matrix $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{pmatrix}$ with respect the standard basis of $V_3(R)$ CO4-App (16)

Or

(b) Let $T: R^2 \to R^2$ by $T(a_1 + a_2, a_1)$, Calculate nullity(T),rank(T), Is T one-to-one and Is T is onto? CO4-App (16)

20. (a) (i) Show that the following function defines an inner product on $V_2(R)$ where $x=(x_1,x_2)$ and $y=(y_1,y_2)$ and $\langle x,y\rangle = 6x_1y_1+7x_2y_2$ CO5-App (8)

(ii) If $x=(2,1+i,i)$ and $y=(2-i,2,1+2i)$ then verify Schwarz's inequality. CO5-App (8)

Or

(b) Show that $V_2(R)$ is an inner product space with inner product defined by $\langle x,y\rangle = x_1y_1+x_2y_1+x_1y_2+4x_2y_2$ where $x=(x_1,x_2)$ *and* $y=(y_1,y_2)$ CO5-App (16)

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