A **Reg. No. :** | | | | | | | | | | | | |

Question Paper Code: R2M08

B.E./B.Tech. DEGREE EXAMINATION, MAY 2024

Second Semester

Computer Science and Business Systems

R21UMA208 - LINEAR ALGEBRA AND NUMERICAL TECHNIQUES

(Regulations 2021)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. If 0,3,4 are the Eigen value of a matrix A then $|A| =$ ______ CO1-App

   (a)12 (b) 0 (c) 3 (d) 4

2. If $A = \begin{pmatrix} a & 1 \\ 3 & b \end{pmatrix}$ has Eigen values of 2,-2 then a and b are_______ CO1-App

   (a) 1,-1 (b) -1,-1 (c) 1,1 (d) 0,1

3. When Gauss Jordan method is used to solve AX=B, A is transferred CO6- U

   (a)Lower triangular (b)Upper triangular (c)Diagonal (d) Unit

4. By Gauss elimination method, solve $x + y = 2$, $2x + 3y = 5$ CO2-App

   (a)1,2 (b) 1,1 (c) 1,0 (d) 0,1

5. Gauss Seidel method iteration converges if the coefficient matrix is _______dominant CO6- U

   (a) Squarely (b) Logically (c) Diagonally (d) Symmetrically

6. Newton forward interpolation formula is used for______ CO6- U

   (a) Open (b) Unequal (c) Equal (d) Closed

7. The trivial subspace of a vector space V are_________ CO6 – U

   (a){0} (b) V (c) W (d){0}and V

8. The dim(R^3) is ________ CO6–U

   (a)1 (b) 2 (c) 3 (d) 0

9. For any two vectors x and y in an inner product space V,find $\|x+y\| \leq$ ____ CO6 - U

(a) $\|x\|+\|y\|$ (b) $\|x\|\|y\|$ (c) $\|x\|-\|y\|$ (d) $\|x\|/\|y\|$

10. x is called a unit vector then find $\|x\| =$ ____ CO6 - U

(a) 0 (b) 1 (c) 2 (d) 3

PART – B (5 x 2= 10 Marks)

11. Find the sum and product of the eigen values of the matrix CO1 -App

$$\begin{pmatrix} -1 & 1 & 1` \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$

12. Apply Gauss-Jordan method calculate the inverse of CO2- App

$$A = \begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix}$$

13. Show that Newton's Raphson formula to find $\sqrt{N}$ can be expressed in the form CO3 -App

$$x_{n+1} = \frac{1}{2}\left[x_n + \frac{N}{x_n}\right].$$

14. Show that $W = \{(a,0,0) / a \in R\}$ is a subspace of R^3 CO4- App

15. If $x = (2, 1+i, i)$ and $y = (2-i, 2, 1+2i)$ .Find $\langle x, y \rangle$ CO5- App

PART – C (5 x 16= 80Marks)

16. (a) Using Cayley Hamilton theorem CO1- App (16)

find A^4 and A^{-1} when $\mathbf{A} = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$.

Or

(b) Reduce the following quadratic forms into canonical form by an orthogonal transformation and find its rank, index, signature and nature $x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$. CO1-App (16)

17. (a) (i) Apply Gauss elimination method to solve CO2-App (8)

$2x+y+z=5, 3x+5y+2z=15, 2x+y+4z=8$.

(ii) Apply Gauss Jordan method to solve CO2-App (8)

$10x+y+z=12$, $2x+10y+z=13$, $x+y+5z=7$

Or

(b) Solve the following using triangularization method CO2-App (16)

$x+y+z=9, 2x-3y+4z=13, 3x+4y+5z=40$

18. (a) (i) Using Lagrange's interpolation formula find f(3) for the following data CO3 -App (8)

X	0	1	2	5
Y	2	3	12	147

(ii) Using Newton's divided difference ,find the value of f(8) from the table CO3 -App (8)

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

Or

(b) Find the numerically largest dominant eigen value of CO3- App (16)

$$A=\begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$$ by power method.

19. (a) (i) Verify the vectors (2,1,0) , (-3,-3,1) ,(-2,1,-1) in R^3 is a basis of R^3 CO4-App (8)

(ii) If T: $R^2 \to R^2$ be linear transformation defined by $T(a_1, a_2) = (a_1 + a_2, a_1)$ then find nullity(T) ,rank(T), Is T one-to-one? Is T onto? Also check the rank nullity theorem CO4-App (8)

Or

(b) (i) Find the dimension of the subspace spanned by the vectors (1,0,2) ,(2,0,1) ,(1,0,1) in $V_3(R)$ CO4-App (8)

(ii) Find the matrix of the linear transformation T: $R^2 \to R^2$ defined by T(a,b) =(2a-3b,a+b) for the standard basis of R^2 CO4-App (8)

20. (a) (i) Show that the following function defines an inner product on $V_2(R)$ where $x=(x_1, x_2)$ and $y=(y_1, y_2)$ and $\langle x, y\rangle = 6x_1y_1 + 7x_2y_2$ CO5-App (8)

(ii) If $x=(2,1+i,i)$ and $y=(2-i,2,1+2i)$ then verify Schwarz's inequality. CO5-App (8)

Or

(b) Apply Gram-Schmidt process to construct an orthonormal basis for $V_3(R)$ with the standard inner product for the basis $\{v_1, v_2, v_3\}$ where $v_1=(1,0,1)$, $v_2=(1,3,1)$ and $v_3=(3,2,1)$ CO5-App (16)