A **Reg. No. :** | | | | | | | | | | | | |

Question Paper Code: R1M02

B.E./B.Tech. DEGREE EXAMINATION, APRIL 2024

First Semester

R21UMA102- MATRIX & CALCULUS

(Common to ALL branches except CSBS)

(Regulations R2021)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The Index of the matrix A is the number of _____ Eigen values of A CO6- U

   (a) zero (b) non zero (c) negative (d) positive

2. Nature of the Canonical form $f(x_1, x_2,) = 2x_1^2 + x_2^2$ CO6- U

   (a) positive definite (b) negative definite (c) positive semi definite (d) indefinite

3. The $(n+1)^{th}$ derivative of x^n CO6- U

   (a) $n!$ (b) $(n+1)!$ (c) n (d) 0

4. The n^{th} derivative of x^{2n} CO6- U

   (a) $(2n)!$ (b) $2n$ (c) $n!$ (d) None

5. The stationary points of $x^2 - xy + y^2 - 2x + y$ is _____ CO3 - App

   (a) (-1, 0) (b) 0, -1) (c) (1, 0) (d) (0, 1)

6. If $AC-B^2>0$ and $A<0$, then f(a,b) has a _____ value. CO6 - U

   (a) minimum (b) maximum (c) not extremum (d) inconclusive

7. $\int_0^{\infty} e^{-x} x^4 dx =$ CO4 - App

   (a) 4 (b)4! (c) 5 (d) 5!

8. Value of $\Gamma(6)$ is equal to **CO4 - App**

(a) 5! (b) 4 (c) 5 (d) 6

9. $\int_0^1 \int_0^2 \int_0^3 dxdydz$ is equal to **CO5 - App**

(a) 2 (b) 3 (c) 4 (d) 6

10. The region of integration of the integral $\int_0^1 \int_0^x f(x,y)dxdy$ is **CO6 - U**

(a) square (b) rectangle (c) triangle (d) circle

PART – B (5 x 2= 10 Marks)

11. Find the Eigen values of A^{-1} and $A^2 + 3I$ for the matrix CO1 - App

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 5 \end{bmatrix}$$

12. Evaluate $\lim_{x \to 1} \left(\frac{4x^3 - 1}{x - 1} \right)$ CO2- App

13. If u = F (x-y, y-z, z-x), prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ CO3 -App

14. Find the value of CO4 - App

$$\int_0^\infty x^7 e^{-x} dx$$

15. Sketch the region of integration CO5 - App

$$\int_0^1 \int_0^x f(x,y)\,dy\,dx$$

PART – C (5 x 16= 80Marks)

16. (a) (i) Find the Eigen values and Eigen Vectors of CO1 - App (8)

$$\begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

(ii) Using Cayley-Hamilton theorem find CO1 - App (8)

A^{-1} for $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 5 & 4 \\ -1 & 3 & 2 \end{bmatrix}$

Or

(b) Reduce the Q.F $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ to a canonical form by an orthogonal transformation and hence find rank, signature, index and nature — CO1 - App — (16)

17. (a) (i) Suppose that a corpse was discovered in a hotel room at midnight and its temperature was 80°F. The temperature of the room is kept constant at 60°F. Two hours later the temperature of the corpse dropped to 75°F. Find the time of death. — CO2 - Ana — (8)

(ii) Expand $e^{\cos x}$ by Maclaurin's series up to the term containing x^4 — CO2 - App — (8)

Or

(b) (i) The radioactive Isotope Indium-III is often used for diagonosis and imaging in nuclear medicine. Its half-life is 2.8days. What was the initial mass of the isotope before decay, if the mass in 2 weeks was 5g. — CO2 -Ana — (8)

(ii) Find the nth derivative of $\frac{x}{2x^2 + 3x + 1}$ — CO2 - App — (8)

18. (a) (i) Find the dimensions of rectangular box without top of maximum capacity with surface area 432 square meters — CO3 -App — (10)

(ii) If $u = a\cosh x\cos y$, $v = a\sinh x\sin y$ .then show that $\frac{\partial(u,v)}{\partial(x,y)} = \frac{1}{2}a^2(\cosh 2x - \cos 2y)$ — CO3 -App — (6)

Or

(b) (i) Using Taylor's series expand $e^x \sin y$ about $\left(1, \frac{\pi}{2}\right)$ up to third degree terms. — CO3 -App — (8)

(ii) Obtain the extreme values of the function $f(x,y) = x^3y^2(1 - x - y)$ — CO3 -App — (8)

19. (a) (i) Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ — CO4 -App — (10)

(ii) Compute $\int_0^{\frac{\pi}{2}} \frac{1}{1 + \cot x}\,dx$ — CO4 -App — (6)

Or

(b) (i) Determine the reduction formula for $\int \cos^n x dx$ CO4 -App (8)

(ii) Prove that $\beta\left(m, \frac{1}{2}\right) = 2^{2m-1}\beta(m, m)$ CO4 -App (8)

20. (a) Using the Triple integration, compute the volume of the Sphere $x^2 + y^2 + z^2 = a^2$ CO5 -App (16)

Or

(b) (i) Changing into polar coordinates and hence evaluate CO5 -App (8)

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \left(x^2 + y^2\right) dy dx$$

(ii) Using the double integration, compute the area of the circle $x^2 + y^2 = a^2$ CO5 -App (8)