**A** **Reg. No. :** | | | | | | | | | | | | |

## Question Paper Code: U1M03

B.E./B.Tech. DEGREE EXAMINATION, APRIL 2024

First Semester

Computer Science and Business systems

21UMA103- PROBABILITY AND INFERENTIAL STATISTICAL TECHNIQUES

(Regulations 2021)

(Statistical Tables may be permitted)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The range of probability of an event is CO6- U

   (a) $0 \leq P(E) \leq 1$ (b) $0 \leq P(E) \leq 2$ (c) $1 \leq P(E) \leq 2$ (d) None of these

2. If A and B are mutually exclusive events, then P(A or B) is CO6 -U

   (a) P(A).P(B) (b) P(A) – P(B) (c) P(A) + P(B) (d) None of these

3. A random variable X is uniformly distributed between 3 and 11. Find the mean of X. CO2 - App

   (a) 12 (b) 9 (c) 7 (d) 8

4. Which of the following discrete distributions follow memory less property: CO6 -U

   (a) Geometric (b) Gamma (c) Normal (d) None of these

5. The joint probability density function is $f(x, y) = k$, $0 < x < 2$, $0 < y < 1$. Estimate K = CO3 - App

   (a) 4 (b) 1 (c) ½ (d) 2

6. Var (2X + 3) = _____ CO3 - App

   (a) 4Var (X) + 9Var (Y) (b) 4Var (X) (c) 9Var (Y) (d) 0

7. The range of 16, 18, 18, 16, 18, 20, 17, 19, 16, 24. CO4 -App

   (a) 12 (b) 8 (c) 9 (d) 10

8. If the mean of first n natural numbers is 5n / 9, then n = ? CO4 -App

(a) 7 (b) 9 (c) 6 (d) 10

9. The degrees of freedom for chi square tests to fitting a binomial distribution CO6 – U

(a) n – 1 (b) n – 2 (c) n – 3 (d) n – 4

10. F – test is used to test for equality of _____ CO6 - U

(a) Mean (b) Variance (c) Both (a) & (b) (d) None of these

PART – B (5 x 2= 10Marks)

11. Find the mean for the discrete RV X with probability distribution CO1 - App

| x | -2 | -1 | 0 | 1 |
|---|---|---|---|---|
| P(X) | 0.2 | 3k | 0.2 | 0.3 |

12. Calculate the MGF of the RV X whose PDF CO2 – App

P( X = x ) = $\frac{1}{2^x}$, $x = 0,1,2,3.....$

13. If Correlation coefficient $\gamma = 0.4, \sigma_x = 5, \sigma_y = 2$, find the regression coefficient of y on x. CO3 – App

14. Two series A and B with equal means have standard deviations 9 and 10 respectively, which series is more consistent CO4 – App

15. A standard sample of 200 tins of coconut oil gave an average weight of 4.95 kg with a standard deviation of 0.21 kg. Do we accept that the net weight is 5 kg per tin at 5% level of significance? CO5 – App

PART – C (5 x 16= 80Marks)

16. (a) (i) A RV X has the following distribution CO1 – App (8)

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| P(X) | 0 | k | 2k | 2k | 3k | $k^2$ | $2k^2$ | $7k^2+k$ |

(i) Find the value of 'k'

(ii) Find $P(X < 6)$, P $[1.5 < X < 4.5 \ / \ X > 2]$

(ii) The cumulative distribution function of a random variable X is $F(x) = 1-(1+x)e^{-x}, x > 0$. Find the probability density function of X, mean and variance. CO1 – App (8)

Or

(b) (i) A R.V X has the PDF $f(x)=\begin{cases}\frac{1}{3}e^{-\frac{x}{3}}, & x\geq 0\\ 0, & x<0\end{cases}$ CO1 – App (8)

Find

(i) $\boldsymbol{P[X>3]}$

(ii) mean and variance.

(ii) For the following density function CO1 – App (8)

$f(x)=ae^{|-x|}, -\infty<x<\infty$

(i) Find the value of 'a'

(ii) Find mean and variance

17. (a) (i) Explain the M.G.F of Geometric distribution and hence find mean and variance. CO2 –App (8)

(ii) The mileage which car owners get with a certain kind of radial tire is a random variable having an exponential distribution with mean 80,000 km. Derive the probabilities that one of these tires will last (i) at least 30,000 km and (ii) at most 40,000 km CO2 –App (8)

Or

(b) (i) A random variable X has a uniform distribution over (-4, 4) compute CO2 –App (8)

(i) P(X < 2) (ii) P(|X| < 3) and (iii) P(X > 1).

(ii) Explain the M.G.F of Binomial distribution and hence find mean and variance. CO2 –App (8)

18. (a) (i) The two dimensional RV (X,Y) has the density function CO3 –App (8)

$f(x,y)=\frac{x+2y}{27}$ x = 0, 1, 2; y = 0, 1, 2.

Find (i) The Marginal distribution function of X and Y

(ii) Find the Conditional distribution of Y for X = 1

(ii) Obtain the Correlation coefficient for the following data CO3 –App (8)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| X | 12 | 15 | 17 | 18 | 23 | 16 | 25 | 27 |
| Y | 110 | 120 | 124 | 130 | 136 | 122 | 140 | 143 |

Or

(b) (i) The joint probability mass function of (X,Y) is given by $P(x,y) = k(2x+3y)$ , $x = 0, 1, 2$; $y = 1, 2, 3$. Find marginal distribution function and conditional distribution. CO3 –App (8)

(ii) The joint pdf $f(x,y) = \begin{cases} \frac{8xy}{9}, & 0 \le x \le y \le 2 \\ 0 & otherwise \end{cases}$. CO3 –App (8)

Find

(*i*) $f_X(x)$

(*ii*) $f_Y(y)$

(iii) conditional density function of X given Y.

19. (a) (i) Compute the Median of the following table: CO4 –App (8)

| Marks | 0 – 6 | 6-12 | 12-18 | 18-24 | 24-30 | 30-36 |
|---|---|---|---|---|---|---|
| No. of students | 12 | 17 | 20 | 25 | 14 | 6 |

(ii) Calculate the arithmetic mean of the following table: CO4 –App (8)

| Marks | 0 – 10 | 10 – 20 | 20 -30 | 30 -40 | 40- 50 | 50 -60 |
|---|---|---|---|---|---|---|
| No. of students | 23 | 25 | 22 | 20 | 33 | 25 |

Or

(b) (i) Find the value of x , when mode is 67 CO4 –App (8)

| Marks | 40-50 | 50-60 | 60-70 | 70-80 | 80-90 |
|---|---|---|---|---|---|
| No. of students | 5 | X | 15 | 12 | 7 |

(ii) Compute the Variance of the following data: CO4 –App (8)

| Marks | 0 -5 | 5 - 10 | 10 -15 | 15 - 20 | 20 - 25 | 25 - 30 |
|---|---|---|---|---|---|---|
| No. of students | 5 | 12 | 21 | 22 | 13 | 10 |

20. (a) (i) Two random samples gave the following results: CO5 –App (8)

| Samples | Size | Sample Mean | Sum of the squares of deviation from the mean |
|---|---|---|---|
| 1 | 10 | 15 | 90 |
| 2 | 12 | 14 | 108 |

Examine whether the samples come from the same normal population

(ii) Sandal powder is packed into packets by a machine. A random sample of 12 packets is drawn and their weights are found to be (in kg) 0.49, 0.48, 0.47, 0.48, 0.49, 0.50, 0.51, 0.49, 0.48, 0.50, 0.51 and 0.48. Test if the average weight of the packing can be taken as 0.5 kg CO5 –App (8)

Or

(b) (i) Two researchers adopted different sampling techniques while investigating the same group of students to find the number of students falling into different intelligence level. The results are as follows CO5 –App (8)

| Research | Below Average | Average | Above Average | Excellent |
|---|---|---|---|---|
| X | 86 | 60 | 44 | 10 |
| Y | 40 | 33 | 25 | 2 |

(ii) Two independent samples of sizes 9 and 7 from a normal population had the following values of the variables. CO5 –App (8)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Sample I | 18 | 13 | 12 | 15 | 12 | 14 | 16 | 14 | 15 |
| Sample II | 16 | 19 | 13 | 16 | 18 | 13 | 15 | -- | -- |

Do the estimates of the population variance differ significally at 5% level?