A **Reg. No. :** | | | | | | | | | | | | |

Question Paper Code: U1M02

B.E./B.Tech. DEGREE EXAMINATION, MAY 2024

First Semester

Civil Engineering

21UMA102- MATRIX AND CALCULUS
(Common to ALL branches)

(Regulation 2021)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The sum and product of the Eigen values of CO6- U

$A=\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ are _____

(a) 4, -4 (b) 0, -4 (c) 4, 2 (d) 5, 3

2. The equation $|A-\lambda I|=0$ is called the_____ of the matrix A. CO6- U

(a) Characteristic equation (b) Characteristic polynomial

(c) Eigen value (d) None of the above

3. $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} =$ ___ CO6- U

(a) θ (b) 2 (a) θ (b) 2

4. Derivative of the constants term is CO6- U

(a) 2 (b) $\log a$ (c) 3 (d) 0

5. The degree of the homogeneous function CO6- U

$u=\frac{x^2+y^2}{\sqrt{x}+\sqrt{y}}$ is _____

(a) 2 (b)1 (c) 3/2 (d) 0

6. A point at which $f(x,y)$ has neither maximum nor minimum is called CO3- U

(a) Saddle point (b) Stationary point (c) Maximum point (d) Minimum point

7. $\int (ax+b)^n\,dx$ CO4- U

(a) $\frac{(ax+b)^{n+1}}{a(n+1)}$ (b) $\frac{(ax+b)^{n-1}}{a(n-1)}$ (c) $(ax+b)^n$ (d) $\frac{(ax+b)^n}{an}$

8. Value of $\Gamma\left(\frac{3}{2}\right)$ = _____ CO4- App

(a) $\frac{3}{2}\left|\frac{1}{2}\right.$ (b) $\frac{\pi}{2}$ (c) $\frac{1}{2}$ (d) $\frac{\sqrt{\pi}}{2}$

9. If x = r cosθ, y = r sinθ then dxdy = ____ CO6- U

(a) r drdθ (b) drdθ (c) r^2drdθ (d) $\frac{1}{r}$ drdθ

10. The region of integration of the integral CO6- U

$\int_0^1 \int_0^x f(x,y)dxdy$ is

(a) square (b) rectangle (c) triangle (d) circle

PART – B (5 x 2= 10 Marks)

11. Find the constants a and b such that the matrix CO6- U

A = $\begin{pmatrix} \mathbf{a} & \mathbf{4} \\ \mathbf{1} & \mathbf{b} \end{pmatrix}$ has 3 and -2 as its Eigen values

12. Find n^{th} *derivative of* $\sin x \cos 3x$ CO2- U

13. If u= $\frac{y^2}{x}$, v = $\frac{x^2}{y}$ find $\frac{\partial(x,y)}{\partial(u,v)}$ CO3 -App

14. Calculate $\Gamma\left(\frac{7}{2}\right)$ CO4- App

15. Evaluate $\int_0^1 \int_0^1 \mathbf{x^2 ydydx}$ CO5- App

PART – C (5 x 16= 80 Marks)

16. (a) Verify Cayley-Hamilton theorem and hence find $\mathbf{A^{-1}}$ **and** $\mathbf{A^4}$ **for** $\mathbf{A}=\begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ CO1- App (16)

Or

(b) Reduce the Q.F $\mathbf{2}\,xy + \mathbf{2}yz + \mathbf{2}$zx to a canonical form by an orthogonal transformation and hence find rank, signature, index and nature. CO1- App (16)

17. (a) (i) A body originally at 80^0C cools down to 60^0C in 20 minutes, the temperature of the air being 40^0C . What will be the temperature of the body after 40 minutes from the original? CO2- Ana (8)

(ii) Expand $e^{\sin x}$by Maclaurin's series up to the term containingx^4 CO2- App (8)

Or

(b) (i) If 30% of radioactive substance disappeared in 10 days, how long will it take for 90% of it to disappear? CO2- Ana (8)

(ii) If $y = e^{ax}\cos bx$, prove that $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2 + b^2)y = 0$ CO2- Ana (8)

18. (a) The temperature $u(x,y,z)$ at any point in space is $u = 400xyz^2$. Find the highest temperature on surface of the sphere $x^2 + y^2 + z^2 = 1$. CO3- Ana (16)

Or

(b) (i) Find the extreme values of $\mathbf{x^3 + y^3 - 3x - 12y + 20}$ CO3- App (8)

(ii) Expand as Taylor's series $e^x \log(1+y)$about (0,0) up to third degree term. CO3- App (8)

19. (a) (i) Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ CO4- App (8)

(ii) Compute CO4- App (8)

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{(\cos x)} + \sqrt{(\sin x)}} dx$$

Or

(b) Evaluate CO4- App (16)

$\int_0^{\frac{\pi}{2}} \cos^m x \sin^n x\, dx$

20. (a) Using the Triple integration, compute the volume of the tetrahedron bounded by the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ and the co-ordinate plane $x=0, y=0, z=0$ — CO5- App — (16)

Or

(b) (i) Show that the area between the parabola $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}\mathbf{a}^2$ — CO5- App — (8)

(ii) Change the order of integration and hence evaluate $\int_0^a \int_x^a \left(x^2+y^2\right) dy dx$ $\int_0^{4\mathbf{a}} \int_{\frac{\mathbf{x}^2}{4\mathbf{a}}}^{2\sqrt{\mathbf{ax}}} \mathbf{xydydx}$ — CO5- App — (8)