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Reg No. : | | | | | | | | | | | | |

Question Paper Code: 91002

B.E./B.Tech. DEGREE EXAMINATION, MAY 2024

First Semester

Civil Engineering

19UMA102- ENGINEERING MATHEMATICS I

(Common to ALL branches)

(Regulation 2019)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. If $A = \begin{pmatrix} \mathbf{a} & \mathbf{1} \\ \mathbf{3} & \mathbf{b} \end{pmatrix}$ has Eigen values 2 and -2, then a and b are ___ CO6- U

(a) 1, -1 (b) -1, -1 (c) 1, 1 (d) -1, 1

2. If 0,3,4 are the Eigen values of a matrix A then $|A| =$___ CO6- U

(a) 0 (b) 1 (c) 2 (d) 3

3. $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} =$ ___ CO6- R

(a) θ (b) 2 (c) 1 (d) 0

4. The Radioactive decay law states that _____ CO6- R

(a) $\frac{dN}{dt} = \lambda N$ (b) $\frac{dN}{dt} = -\lambda N$ (c) $\frac{dN}{dt} = N$ (d) $\frac{dN}{dt} = -\lambda / N$

5. If u, v, w are functionally dependent functions of independent variables x, y, z then $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ = _____ CO6- U

(a) $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ (b) 0 (c) 1 (d) -1

6. The stationary points of $x^2 - xy + y^2 - 2x + y$ is _____ CO3- App

(a) (-1, 0) (b) 0, -1) (c) (1, 0) (d) (0, 1)

7. The value of $\Gamma(\frac{1}{2})$ = _____ CO6- U

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\sqrt{\pi}}{2}$ (d) $\sqrt{\pi}$

8. $\int_0^{\frac{\pi}{2}} \sin^6 x\, dx$ is _____ CO4- App

(a) $\frac{\pi}{85}$ (b) $\frac{3}{256}$ (c) $\frac{\pi}{32}$ (d) $\frac{5\pi}{32}$

9. $\int_0^1 \int_0^2 \int_0^3 dxdydz$ is equal to CO5- App

(a) 2 (b) 3 (c) 4 (d) 6

10. The area of an ellipse is CO6- R

(a) πr^2 (b) πr (c) πab (d) π

PART – B (5 x 2= 10 Marks)

11. Find the constants a and b such that the matrix A = $\begin{pmatrix} a & 4 \\ 1 & b \end{pmatrix}$ has 3 and -2 as its Eigen values CO6- U

12. Find n^{th} *derivative of* $\sin x \cos 3x$ CO6- U

13. If $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$ *find* $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$ CO3- App

14. Find the value of $\int_0^1 x^8 (1-x)^5 dx$ CO4- App

15. Evaluate $\iint xydxdy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$ CO5- App

PART – C (5 x 16= 80 Marks)

16. (a) Using Characteristic equation to the Eigen values and Eigen vectors of $\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$ CO1- App (16)

Or

(b) Reduce the Q.F $x_1^2 + 5x_2^2 + x_3^2 + 2x_1x_2 + 2x_2x_3 + 6x_3x_1$ to a canonical form by an orthogonal Transformation and hence find rank, signature, index and nature. CO1- App (16)

17. (a) (i) Expand $e^{\sin x}$ by Maclaurin's series up to the term containing x^4 CO2- App (8)

(ii) A pot of boiling water 100°C is removed from the fire and allowed to cool at 30°C room temperature. Two minutes later, the temperature of the water in the pot is 90°C. What will be the temperature of the water after 5 minutes? CO2- App (8)

Or

(b) (i) Find n^{th} derivatives of $\frac{1}{(x^2 - a^2)}$ CO2- App (8)

(ii) The initial mass of an Iodine isotope was 200g. Determine the Iodine mass after 30 days if the half-life of the isotope is 8 days? CO2- Ana (8)

18. (a) (i) Find the maximum volume of the rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. CO3- App (8)

(ii) Compute the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 if $y_1 = \frac{x_2x_3}{x_1}$, $y_2 = \frac{x_3x_1}{x_2}$, $y_3 = \frac{x_1x_2}{x_3}$ CO3- App (8)

Or

(b) (i) Find the extreme values of $x^3 + y^3 - 3x - 12y + 20$ CO3- App (8)

(ii) Expand as Taylor's series $e^x \log(1+y)$ about (0,0) up to third degree term. CO3- App (8)

19. (a) (i) Compute $\int_0^{\frac{\pi}{2}} \frac{(\sin x)^{\frac{3}{2}}}{(\cos x)^{\frac{3}{2}} + (\sin x)^{\frac{3}{2}}} dx$ CO4- App (8)

(ii) Prove that $\beta\left(m, \frac{1}{2}\right) = 2^{2m-1}\beta(m, m)$ CO4- App (8)

Or

(b) Derive the relationship between Beta and Gamma functions CO4- App (16)

20. (a) (i) Using the Triple integration, compute the volume of the Sphere $\mathbf{x^2 + y^2 + z^2 = a^2}$ CO5- App (8)

(ii) Using the double integration, compute the area of the ellipse $\frac{\mathbf{x^2}}{\mathbf{a^2}} + \frac{\mathbf{y^2}}{\mathbf{b^2}} = \mathbf{1}$ CO5- App (8)

Or

(b) (i) Using the Triple integration, compute the volume of the Ellipsoid $\frac{\mathbf{x^2}}{a^2} + \frac{\mathbf{y^2}}{b} + \frac{\mathbf{z^2}}{c^2} = 1$ CO5- App (8)

(ii) Change the order of integration and hence evaluate CO5- App (8)

$$\int_{0}^{4\mathbf{a}} \int_{\frac{\mathbf{x}^2}{4\mathbf{a}}}^{2\sqrt{\mathbf{ax}}} \mathbf{xydydx}$$