A **Reg. No. :** | | | | | | | | | | | | |

Question Paper Code: U3025

B.E./B.Tech. DEGREE EXAMINATION, MAY 2024

Third Semester

Civil Engineering

21UMA325- PROBABILITY, STATISTICS AND TRANSFORM TECHNIQUES

(Regulations 2021)

Duration: Three hours Maximum: 100 Marks

Answer All Questions

PART A - (10x 1 = 10 Marks)

1. Probability of an impossible event is CO6-U

   (a) 1 (b) 10 (c) 0 (d) 100

2. The mean of the random variable is denoted by CO6- U

   (a) E(X) (b) $E(X^2)$ (c) 0 (d) 1

3. The degrees of freedom in t-tests is CO6- U

   (a) n-1 (b) n-2 (c) n-3 (d) n-4

4. Choose the t-test for mean CO6- U

   (a) $t = \frac{\overline{x_1} - \mu}{s / \sqrt{n-1}}$ (b) $t = \frac{\overline{x_1} + \mu}{s / \sqrt{n-1}}$ (c) t = 0 (d) None of the above

5. If $f(x + t) = f(x)$, then f(x) is said to be an _____ CO6- U

   (a) Odd Function (b) Even Function (c) Periodic function (d) Self Reciprocal

6. The Fourier constant b_n in $(-\pi,\pi)$ for x sin x is _____ CO6- U

   (a) x^2 (b) 3x (c) **0** (d) 1

7. $F[xf(x)]$ = _____ CO6- U

   (a) $-F_c[f(x)]$ (b) $-\frac{d}{ds}\{F_s[f(x)]\}$ (c) $-F_s[f(x)]$ (d) $-\frac{d}{ds}\{F_c[f(x)]\}$

8. $F_s[e^{-ax}] =$ _____ CO6- U

(a) $\sqrt{\frac{2}{\pi}}\frac{s}{s^2+a^2}$ (b) $\sqrt{\frac{2}{\pi}}\frac{a}{s^2+a^2}$ (c) $\sqrt{\frac{2}{\pi}}\frac{a^2}{s^2+a^2}$ (d) $\sqrt{\frac{2}{\pi}}\frac{s^2}{s^2+a^2}$

9. The Z transform of a unit step function is _____. CO5- U

(a) $\log(\frac{z}{z+1})$ (b) $\frac{z}{z+1}$ (c) $\log(\frac{z}{z-1})$ (d) $\frac{z}{z-1}$

10. Evaluate $Z(\frac{1}{n!})$ CO5- U

(a) $e^{-1/z}$ (b) $e^{1/z}$ (c) e^{2z} (d) $e^{1/z-2}$

PART – B (5 x 2= 10Marks)

11. If $f(x) = \begin{cases} Kxe^{-x}, & x > 0 \\ o, & elsewhere \end{cases}$ is the PDF of a RV X, Find K CO1- App

12. Write the important properties of F-distribution CO2- U

13. Find b_n in the Fourier series of $f(x) = |\cos x|$ in $(0,2\pi)$. CO3- App

14. State Fourier integral theorem. CO6- U

15. Prove that $Z(sin\frac{n\pi}{2})=\frac{z}{z^2+1}$ CO5 App

PART – C (5 x 16= 80Marks)

16. (a) A Random Variable X has the following probability distribution CO1-App (16)

X=x	0	1	2	3	4	5	6	7
P(X=x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Find (i) 'k'

(ii) $P(X<6), P(X\geq 6)$ & $P(1.5<X<4.5/X>2)$

(iii) If $P(X\leq k)>1/2$ find the minimum value of 'k'

(iv) Distribution function of x

Or

(b) (i) The number of monthly breakdowns of a computer is a R.V. having a Poisson distribution with mean equal to 1.8. Find the Probability that his computer will function for a month (a) Without a breakdown (b) With only one breakdown (c) With at least one breakdown CO1 -App (8)

(ii) Using an Geometric distribution State and Prove the memory less property. CO1 -App (8)

17. (a) (i) A sample analysis of examination results of 500 students was made. It was found that 220 students have failed, 170 have secured a third class, 90 have secures a second class and the rest, a first class. So these figures support the general belief that the above categories are in the ratio 4:3:2:1 respectively? CO2 -Ana (8)

(ii) Two group of students A and B were tested , the marks obtained were as follows CO2 -Ana (8)

A	18	20	36	50	49	36	34	49	41
B	29	28	26	35	30	44	46		

Examine the significance of difference between the average marks secured by the students of the above two groups

Or

(b) (i) Two horses A and B were tested according to time (in seconds) to run on a particular track with the following results: CO2 -Ana (8)

Horse A	28	30	32	33	33	29	34
Horse B	29	30	30	24	27	29	

Test whether horse A is running faster than B at 5% level.

(ii) A group of 10 rats fed on diet A and another group of 8 rats fed on diet B, recorded the following increase in weight. CO2 -Ana (8)

Diet A	5	6	8	1	12	4	3	9	6	10
Diet B	2	3	6	8	10	1	2	8		

Find the variances are significantly different

18. (a) Express $f(x) = \frac{1}{2}(\pi - x)$as a Fourier series of period 2π in the interval $0 < x < 2\pi$. CO3- App (16)

Or

(b) The table of values of the function $y = f(x)$ is given below: CO3- App (16)

x:	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y:	1.8	0.3	0.5	2.16	1.3	1.76	1.8

Find a Fourier series up to the third harmonic to represent $f(x)$in terms of x

19. (a) Compute the Fourier Transform of $f(x)=\begin{cases}1-x^2 & |x|<1\\ 0 & otherwise\end{cases}$ CO4-App (16)

and hence evaluate the value of (i) $\int_0^\infty \frac{\sin t - t\cos t}{t^3}dt$

$\int_0^\infty \left(\frac{\sin t - t\cos t}{t^3}\right)^2 dt$

Or

(b) Compute (i) $\int_0^\infty \frac{dx}{(x^2+1)(x^2+4)}$ (ii) $\int_0^\infty \frac{x^2 dx}{(x^2+9)^2}$ using Fourier transform CO4-App (16)

20. (a) (i) Solve the difference equation $y_{n+2}+6y_{n+1}+9y_n = 2^n$ given that $y_0 = 0,\ y_1 = 0$ CO5- App (8)

(ii) Using Convolution theorem find $Z^{-1}\left[\frac{8z^2}{(4z-3)(2z+1)}\right]$ CO5- App (8)

(b) (i) Solve the difference equation $y_{n+2}+4y_{n+1}+3y_n = 2^n$ given that $y_0 = 0,\ y_1 = 0$ CO5- App (8)

(ii) Evaluate $Z[\cos n\theta]$ *and* $Z[\sin n\theta]$ CO5- App (8)