A **Reg. No. :** | | | | | | | | | | |

Question Paper Code: 53021

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2024

Third Semester

Civil Engineering

15UMA321 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to EEE, ECE, EIE, MECH, Chemical, Biomedical and

Agriculture Engineering Branches)

(Regulation 2015)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The constant term in the Fourier series is CO1- R

   (a) a_0 (b) b_1 (c) a_5 (d) b_4

2. The root mean square value of f(x) = x in (0, 1) interval CO1- R

   (a) 2/3 (b)$1/(3)^{1/2}$ (c) $2/(3)^{1/2}$ (d)4/5

3. Fourier integral of f(x) = 1. $0 < x < \infty$ CO2- R

   (a) 0 (b)1 (c) Not defined (d) Very large number

4. Give a function which is self reciprocal under sine transform CO2- R

   (a) x (b) x^2 (c) $1/(x)^{(1/2)}$ (d) $1/(x)^{(3/2)}$

5. Find $Z\left[a^{n-1}\right]$ CO3- R

   (a) $\frac{az}{z-1}$ (b) $\frac{1}{z-1}$ (c) $\frac{z^2}{z-a}$ (d) $1/a\ (\frac{z}{z-a})$

6. Find CO3- R

$Z^{-1}\left[\frac{z}{z+1}\right]$

(a) $(-1)^n$ (b) $(-a)^n$ (c) $(-t)^n$ (d) $(1)^n$

7. The p.d.e of z = ax+by is CO4- R

(a) x+y (b) qx+py (c) px+qy (d) x-y

8. Find the P.I of $[D^2 + 4DD']z = e^x$ CO4- R

(a) 1 (b) e^x (c)0 (d) e^{x-1}

9. What is the constant a^2 in the wave equation CO5- R

(a) $a^2 = \frac{T}{m}$ (b) $a^2 = \frac{1}{m}$ (c) $a^2 = \frac{T}{2}$ (d) $a^2 = \frac{T^2}{m}$

10. Governing equation of two dimensional steady state heat equation is CO5- R

(a) $\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial y^2} = 0$ (b) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 1$ (c) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial y} = 0$ (d) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

PART – B (5 x 2= 10 Marks)

11. Explain Dirichlet's condition CO1- R

12. State the Convolution theorem for Fourier Transforms CO2- R

13. Define Difference equations CO3- R

14. From the p.d.e by eliminating arbitrary constants a and b from CO4- R

$z = (x+a)^2 + (y-b)^2$

15. Write all variable separable solutions of the one dimension heat equation. CO5- R

PART – C (5 x 16= 80Marks)

16. (a) Find the Fourier series of x^2 in $(-\pi, \pi)$. Hence prove the following — CO1- App (16)

$(a)\ \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots=\frac{\pi^2}{6}$

$(b)\ \frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\cdots=\frac{\pi^2}{12}$

$(c)\ \frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\cdots=\frac{\pi^4}{90}$

Or

(b) Determine the first two harmonic of the Fourier series for the following values. — CO1- App (16)

X:	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$
Y:	1.98	1.30	1.05	1.30	-0.88	-0.25

17. (a) Find the Fourier transform of — CO2- App (8)

$$f(x)=\begin{cases}1-|x| & \text{if } |x|<1\\ 0 & \text{if } |x|>1\end{cases}.$$

Hence deduce the following:

$(i)\int_0^\infty\left(\frac{\sin t}{t}\right)^2 dt=\frac{\pi}{2}$ — CO2- App (4)

$(ii)\int_0^\infty\left(\frac{\sin t}{t}\right)^4 dt=\frac{\pi}{3}$ — CO2- App (4)

Or

(b) Show that — CO2- App (16)

$e^{-x^2/2}$ is self reciprocal under Cosine Transform.

18. (a) Find CO3- Ana (4)

(i) Z [$a^n \cos n\theta$]

(ii) Z [$\sin n\theta$] CO3- Ana (4)

(iii) Using convolution theorem, evaluate the inverse Z – transform of $\frac{z^2}{(z-a)(z-b)}$ CO3- Ana (8)

Or

(b) Solve $y_{n+2} + 6\, y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$, using Z – transform. CO3- Ana (16)

19. (a) (i) Find the singular integral of $z = px + qy + p^2 + pq + q^2$ CO4-App (8)

(ii) Solve $p\sqrt{x} + q\sqrt{y} = \sqrt{z}$ CO4-App (8)

Or

(b) Solve $(D^2 + 2DD' + D'^2)z = x^2y + e^{x-y}$. CO4- App (16)

20. (a) A tightly stretched flexible string has its ends fixed at x = 0 and x = ℓ. At time t = 0, the string is given a shape defined by f (x) = $kx(\ell - x)$, where 'k' is constant and then released from rest. Find the displacement of any point 'x' of the string at any time t > 0. CO5- U (16)

Or

(b) An insulated rod of length *l* has its ends A and B maintained at 0^0 C and 100^0 c respectively until steady state conditions prevail. If B is suddenly reduced to 75^0 C and at A raised to 25^0 C, find the temperature at a distance x from A at time t. CO5- U (16)