A **Reg. No. :** | | | | | | | | | | | | |

Question Paper Code: R2M06

B.E./B.Tech. DEGREE EXAMINATION, APRIL 2024

Second Semester

Civil Engineering

R21UMA206- DIFFERENTIAL EQUATIONS, COMPLEX ANALYSIS & TRANSFORM TECHNIQUES

(Regulations R2021)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. $\frac{1}{(D-m)^2}e^{mx}$ = _____ CO1-App

(a) xe^{mx} (b) x^2e^{mx} (c) $\frac{\mathbf{x^2}}{\mathbf{2}}\mathbf{e^{mx}}$ (d) $\frac{\mathbf{x^2}}{m}\mathbf{e^{mx}}$

2. The complementary function of $(4D^2 - 3D - 1)y = 2\sin 2x$ is _____ CO1- App

(a) $Ae^x + \mathbf{Be}^{-\frac{x}{4}}$ (b) $Ae^{-x}+Be^{5x}$ (c) $(A+Bx)e^{2x}$ (d) $Ae^{x}+Be^{4x}$

3. Div $\bar{r}$ = ____ CO6- U

(a) 0 (b) 1 (c) 3 (d) $\bar{r}$

4. Divergence of vector $\mathbf{x^2\vec{i}+y^2\vec{j}+z^2\vec{k}}$ at (1, 2, -3) is _____ CO2- App

(a) 8 (b) 4 (c) -3 (d) 0

5. The PDE obtained from z = (x+a)(y+b) is __. CO3- App

(a) 3z = px + qy (b) py - qx = 0 (c) z = pq (d) px+qy = 0

6. The PDE of all planes having equal intercepts on the X axis and Y axis is _____ CO3- App

(a) p = q (b) p + q = 0 (c) pq = 1 (d) p(q + 1) = q

7. Simple pole is a pole of order _____ CO6- U

(a) 1 (a) 2 (c) 3 (d) 4

8. The value of $\int_C \frac{dz}{z+2}$, c : | z | = 1 is _____ CO4- App

(a) 2πi (b) -2πi (c) 4πi (d) 0

9. L (sin h at) = _____ CO6- U

(a) $\frac{s}{s^2 - a^2}$ (b) $\frac{a}{s^2 - a^2}$ (c) $\frac{s}{s^2 + a^2}$ (d) $\frac{a}{s^2 + a^2}$

10. $L^{-1}(1)$ = _____ CO4- App

(a) t (b) $\frac{1}{t}$ (c) $\frac{1}{s}$ (d) $\delta(t)$

PART – B (5 x 2= 10 Marks)

11. Compute the particular integral for $(D^2 - 2D + 1)y = \cosh x$. CO1 App

12. Compute if $\varphi = x^2 + y^2 + z^2$ at (1, -1, 1). CO2 App

13. Form the PDEby eliminating arbitrary constants from $z = (x + a)^2 + (y + b)^2$. CO3 App

14. Evaluate $\int_c \frac{z}{z-2} dz$ where C is $|z| = 2$ CO4 App

15. Estimate L[t sint] CO5 App

PART – C (5 x 16= 80 Marks)

16. (a) (i) Using method of variation of parameters solve $(D^2 + 4)y = \sec 2x$. CO1-App (8)

(ii) A colony of bacteria is growing exponentially. At time t=0 it has 10 bacteria in it, and at time t=4 it has 2000. At what time will it have 100,000 bacteria? CO1-App (8)

Or

(b) (i) Solve: $(x^2D^2 + xD + 1)y = x \sin(\log x)$ CO1-App (8)

(ii) Solve: $(D^2 - 4D + 3)y = \sin 3x + e^{2x}$ CO1-App (8)

17. (a) Verify Divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - xz)\vec{j} + (z^2 - xy)\vec{k}$ over the rectangular parallelepiped x = 0, x = a, y = 0, y = b, z = 0, z = c. CO2- App (16)

Or

(b) (i) Using Green's theorem, Evaluate $\int_C (3x^2 - 8y^2)\,dx + (4y - 6xy)dy$ where C is the boundary of the region defined by x = 0, y = 0, x + y= 1 in the xy plane. CO2- App (10)

(ii) Prove that $\bar{F} = (x^2 + xy^2)\bar{\imath} + (y^2 + x^2y)\bar{\jmath}$ is irrotational vector and compute the Scalar potential such that $\bar{F} = \nabla\emptyset$. CO2- App (6)

18. (a) (i)) Solve: $(mz - ny)\,p + (nx - lz)\,q = ly - mx$ CO3- App (8)

(ii) Solve $(D^2 - DD' - 3D'^2)z = \sin(x + y) + e^{6x+y}$ CO3- App (8)

Or

(b) (i) Solve $p^2 + q^2 = x^2 + y^2$ CO3- App (8)

(ii) Form a PDE by eliminating arbitrary functions from $z = px + qy + p^2 - q^2$ CO3- App (8)

19. (a) (i) Evaluate CO4- App (8)

$$f(z) = \int_C \frac{\cos \pi z^2 + \sin \pi z^2}{(z-1)(z-2)}\,dz$$ by using Cauchy's Integral formula

where C is $|z| = 3$

(ii) Expand CO4- App (8)

$\frac{z-1}{(z+2)(z+3)}$ as Laurent's series valid in the region $2 < |z| < 3$

Or

(b) Using Contour integration, to prove CO4- App (16)

$$\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + a^2)(x^2 + b^2)}\,dx = \frac{\pi}{a + b}.\ a > b > 0$$

20. (a) (i) Solve the differential equation $\frac{d^2y}{dt^2} + y = \sin 2t\ ; y(0) = 0\ ; y'(0) = 0$ by using Laplace transform method. CO5- App (10)

(ii) Compute the Laplace Transforms of $\frac{\cos at - \cos bt}{t}$ CO5- App (6)

Or

(b) (i) Using the periodic function, Compute the Laplace transform of CO5- App (8)

$$f(t)=\begin{cases} k\ , & 0\le t\le a \\ -k\ , & a\le t\le 2a \end{cases}$$ and f(t+2a) = f(t).

(ii) Using Convolution Theorem, Compute CO5- App (8)

$$L^{-1}\left[\frac{1}{(s^2+a^2)(s^2+b^2)}\right]$$