A **Reg. No. :** | | | | | | | | | | | | |

Question Paper Code: U2M06

B.E./B.Tech. DEGREE EXAMINATION, MAY 2024

Second Semester

Civil Engineering

21UMA206- DIFFERENTIAL EQUATIONS, COMPLEX ANALYSIS & TRANSFORM TECHNIQUES

(Regulations 2021)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The solution of $(D^3 + D^2 - D - 1)y = 0$ is _____ CO1-App

   (a) $Ae^x + Bxe^x + Cx^2e^x$ (b) $(Ax + B)e^x + C e^{-x}$

   (c) $e^{-x} + (\cos 2x + i \sin 2x)$ (d) $(Ax + B)e^{-x} + C e^x$

2. The complementary function of $(4D^2 - 3D - 1)y = 2 \sin 2x$ is _____ CO1-App

   (a) $Ae^x + Be^{-\frac{x}{4}}$ (b) $Ae^{-x} + Be^{5x}$ (c) $(A+Bx)e^{2x}$ (d) $Ae^x + Be^{4x}$

3. If $\varphi = x^2 + y^2 - z - 10$ then $|\nabla \varphi|$ at (1, 1, 1) is _____ CO2-App

   (a) $2(\vec{i} + \vec{j} + \vec{k})$ (b) $2\vec{i} + 2\vec{j} - \vec{k}$ (c) 3 (d) 9

4. If φ is a vector point function then Curl (gradφ) = CO2-App

   (a) 1 (b) 0 (c) 2 (d) None of the above

5. The PDE obtained from z = (x+a)(y+b) is __. CO3- App

   (a) 3z = px + qy (b) py - qx = 0 (c) z = pq (d) px+qy = 0

6. The PDE of all planes having equal intercepts on the X axis and Y axis is _____ CO3- App

   (a) p = q (b) p + q = 0 (c) pq = 1 (d) p(q + 1) = q

7. Simple pole is a pole of order _____ CO6-U

   (a) 1 (a) 2 (a) 3 (a) 4

8. The poles of z cot z is _____ CO6-U

(a) 0 (b) $\pm n\pi$ (c) 1 (d) π

9. L (sin h at) = _____ CO6-R

(a) $\frac{s}{s^2 - a^2}$ (b) $\frac{a}{s^2 - a^2}$ (c) $\frac{s}{s^2 + a^2}$ (d) $\frac{a}{s^2 + a^2}$

10. $L^{-1}\left(\frac{1}{(s+2)^3}\right)$ = _____ CO6-R

(a) $t\,e^{2t}$ (b) $t^2\,e^{-2t}$ (c) $\frac{e^{-2t}t^2}{2!}$ (d) $\frac{e^{-t}t^3}{3!}$

PART – B (5 x 2= 10Marks)

11. Compute the particular integral for $(D^2 - 2D + 1)y = \cosh x$. CO1-App

12. Compute the unit normal vector to the surface $x^2 + y^2 + z^2 = 1$ at (1, 1, 1). CO2-App

13. Compute the complete integral of $p - q = k$ CO3-App

14. State Cauchy's residue theorem. CO6-U

15. Find $L\left[te^{at}\right]$ CO5-App

PART – C (5 x 16= 80Marks)

16. (a) (i) Solve: $(x^2D^2 - xD + 4)y = x^2 \sin(\log x)$. CO1-App (8)

(ii) Solve: $(D^2 + 4D + 3)y = \sin x + e^{2x}$ CO1- App (8)

Or

(b) (i) At the start of an experiment, there are 200 bacteria. If the bacteria follow an exponential growth pattern with rate k = 0.05. What will be the population after 8 hours? How long will it take for the population to double? CO1- App (8)

(ii) Using method of variation of parameters solve $(D^2 + a^2)y = \tan ax$. CO1- App (8)

17. (a) Verify Divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - xz)\vec{j} + (z^2 - xy)\vec{k}$ over the rectangular parallelepiped x = 0, x = a, y = 0, y = b, z = 0, z = c. CO2-App (16)

Or

(b) (i) Using Green's theorem, Evaluate $\int_C (3x^2 - 8y^2)\,dx + (4y - 6xy)dy$ where C is the boundary of the region defined by X = 0, Y = 0, X + Y= 1 in the XY plane. CO2 -App (8)

(ii) Prove that $\bar{F} = (x^2 + xy^2)\bar{\imath} + (y^2 + x^2y)\bar{\jmath}$ is irrotational vector and compute the Scalar potential such that $\bar{F} = \nabla\emptyset$. CO2 -App (8)

18. (a) (i) Solve $\left(D^2 - 6DD' + 5D'^2\right)z = e^{x+y} + \sin(2x+y)$ CO3-App (8)

(ii) Solve $(y - z)p + (z - x)q = (x - y)$ CO3-App (8)

Or

(b) (i) Solve $p^2 + q^2 = x^2 + y^2$ CO3-App (8)

(ii) Form a PDE by eliminating arbitrary functions from $z = px + qy + p^2 - q^2$ CO3-App (8)

19. (a) (i) Evaluate $f(z) = \int_C \frac{\cos \pi z^2 + \sin \pi z^2}{(z-1)(z-2)} dz$ by using Cauchy's Integral formula where C is $|z| = 3$ CO4-App (8)

(ii) Expand $\frac{z-1}{(z+2)(z+3)}$ as Laurent's series valid in the region $2 < |z| < 3$ CO4-App (8)

Or

(b) Using contour integration, to compute the value of $\int_0^{2\pi} \frac{d\theta}{13 + 5\cos\theta}$ CO4-App (16)

20. (a) (i) Solve the differential equation $\frac{d^2y}{dt^2} + 9y = \cos 2t$ if y(0) = 1 & $y\left(\frac{\pi}{2}\right) = -1$ by using Laplace transform method. CO5-App (8)

(ii) Compute the Laplace Transforms of $te^{-2t} \sin t$ CO5-App (8)

Or

(b) (i) Using the periodic function, Compute the Laplace transform of CO5-App (8)

f(t) = $f(t) = \begin{cases} k, & 0 \le t \le a \\ -k, & a \le t \le 2a \end{cases}$

(ii) Using Convolution Theorem, Compute $L^{-1}\left[\frac{1}{(s^2+a^2)(s^2+b^2)}\right]$ CO5-App (8)