A **Reg. No. :** | | | | | | | | | | | | |

Question Paper Code: R2M07

B.E./B.Tech. DEGREE EXAMINATION, APRIL 2024

Second Semester

Agricultural Engineering

R21UMA207- CALCULUS COMPLEX ANALYSIS AND TRANSFORM TECHNIQUES

(Regulations R2021)

(Common to Biomedical and Biotechnology engineering branches)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. $\frac{1}{(D-m)^2}e^{mx} = \_\_\_\_\_$ CO1- App

(a) xe^{mx} (b) x^2e^{mx} (c) $\frac{x^2}{2}e^{mx}$ (d) $\frac{x^2}{m}e^{mx}$

2. $\frac{1}{D^2}(\sin x) = \_\_\_\_$ CO1- App

(a) sin x (b)- cos x (c) cos x (d) tan x

3. IF $\vec{F}$ is Irrotational then $\nabla \times \vec{F} =$ CO2- U

(a) 1 (b) 2 (c) 0 (d) 3

4. If $\overline{F}$ is a conservative field, then $\overline{F}$ is_____ CO2- U

(a) Solenoidal (b) Irrotational (c) 0 (d) None of these

5. The critical point of the transformation $w = z + \frac{1}{z}$ are ____ CO3- App

(a) ± 1 (b) ± 2 (c) $\pm i$ (d) $-i$

6. The transformation w=1/z is known as _____ CO3- U

(a) Rotation (b) reflection (c) translation (d) inversion

7. If f(z) is analytic at all points inside and on a simple closed curve c, then $\int_C f(z)dz$ = ------ CO4- U

(a) 2πi (b) -2πi (c) 4πi (d) 0

8. The poles of CO6- U

$f(z) = \frac{z^2+1}{1-z^2}$ are

(a)1,0 (b)0,0 (c)1,2 (d)-1,1

9. L (sin h at) = _____ CO5- App

(a) $\frac{s}{s^2 - a^2}$ (b) $\frac{a}{s^2 - a^2}$ (c) $\frac{s}{s^2 + a^2}$ (d) $\frac{a}{s^2 + a^2}$

10. L[f(at)] = _____ CO5- App

(a) $F(\frac{s}{a})$ (b) $a\,F(\frac{s}{a})$ (c) $\frac{1}{a}F(\frac{s}{a})$ (d) a F(s)

PART – B (5 x 2= 10Marks)

11. Compute the Particular Integral of $(D^2 - D - 6)y = 3e^{4x}$ CO1- App

12. Show that $\vec{F} = z\vec{i} + xj + y\vec{k}$ is Solenoidal CO2 -App

13. Find the invariant points of $w = z^2$ CO3- App

14. Using Cauchy's integral formula , Evaluate $\int_c \frac{z}{z-2}dz$ where C is $|z| = 1$ CO4- App

15. Compute $L(e^{-2t})$ CO6 U

PART – C (5 x 16= 80Marks)

16. (a) (i) Solve $(D^2 - 3D + 2)y = 2e^x + 2\cos 2x$ CO1- App (8)

(ii) Solve $(x^2D^2 + 4xD + 2)y = x + \frac{1}{x}$ CO1-App (8)

Or

(b) (i) Using method of variation of parameters solve $(D^2 + a^2)y = \sec ax$ CO1-App (8)

(ii) If the population of a country double in 50 years , in how many years will it triple under the assumption that the rate of increase of proportional to the number of inhabitants? CO1-App (8)

17. (a) Verify Stokes theorem for a vector field defined by $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ in the rectangular region in the XOY plane bounded by the lines $x = \pm a$, $y = 0$, *and* $y = b$. CO2-App (16)

Or

(b) Verify Gauss divergence theorem for the vector function $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ over the cube bounded by $x = 0, y = 0, z = 0$ *and* $x = 1, y = 1, z = 1$ CO2-App (16)

18. (a) (i) Determine the analytic function whose imaginary part is $e^x(x\cos y - y\sin y)$ CO3 -App (8)

(ii) Determine the image of $|z - 1| = 1$ under the transformation $w = \frac{1}{z}$ CO3 -App (8)

Or

(b) (i) If w =u(x,y)+iv(x,y) is an analytic function then prove that the curves of the family u(x,y)= a and the curves of the family v(x,y)=b cut orthogonally ,where a and b are constants. CO3- App (8)

(ii) Determine the bilinear transformation that maps the points ∞,i,0 onto 0,i, ∞ respectively. CO3 -App (8)

19. (a) (i) Using Cauchy's integral formula , Evaluate $\int_C \frac{2z+1}{z(z+1)(z-3)}dz$ where C is the circle $|z| = 2$ CO4-App (8)

(ii) Evaluate f(z) $= \frac{7z - 2}{z(z+1)(z-2)}$ in Laurent's series valid in the region $1 < |z + 1| < 3$ CO4-App (8)

Or

(b) Using Contour integration Prove that $\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + a^2)(x^2 + b^2)}dx = \frac{\pi}{a + b}$ $a > b > 0$ CO4-App (16)

20. (a) (i) Solve the differential equation $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = e^{-t}$ with $y(0) = 1$ & $y'(0) = 0$ by using Laplace transform method CO5-App (8)

(ii) Using Convolution Theorem, Compute $L^{-1}\left[\frac{s}{(s^2 + 4)^2}\right]$. CO5-App (8)

Or

(b) (i) Compute the Laplace transform of $f(t) = \begin{cases} k, & 0 \le t \le a \\ -k, & a \le t \le 2a \end{cases}$ CO5-App (8)

and f(t +2a) = f(t)

(ii) Using Convolution Theorem, Compute $L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$ CO5-App (8)