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Reg. No. : | | | | | | | | | | | |

Question Paper Code: U4M24

B.E./B.Tech. DEGREE EXAMINATION, APRIL 2024

Fourth Semester

Bio Medical Engineering

21UMA424 - PROBABILITY AND INFERENTIAL STATISTICS

(Common to Biotechnology)

(Regulations 2021)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. Probability of an impossible event is CO6-U

(a) 1 (b)10 (c)0 (d) ∞

2. Probability of sure event is CO6- U

(a) 0 (b) 1 (c) 2 (d)10

3. If X and Y are independent random variables then CO6- U

(a) f(x,y) = f(x) . f(y) (b) f(x,y) = f(x) + f(y) (c) f(x,y) = 0 (d) None of the above

4. The marginal density function of X is CO6- U

(a) f(y) (b) f(x,y) (c) f(x) (d) f (x/y)

5. If the Random Process {X(t)} with mean has Auto correlation function $R(\tau) = 16 + 9e^{-|\tau|}$ Then the Variance of the process is CO3- App

(a) 16 (b) 25 (c) 6 (d) 9

6. Given $R(\tau) = 25 + \frac{4}{1+6\tau^2}$ What is E[X^2(t)]? CO3- App

(a) 25 (b) 29 (c) 26 (d) 27

7. The system is said to be stable if CO6- U

(a) $\int_{-\infty}^{\infty} h(t)\, dt < \infty$ (b) $\int_{-\infty}^{\infty} h(t)\, dt > \infty$ (c) $\int_{-\infty}^{\infty} h(t)\, dt > 0$ (d) None of the above

8. If $S_{XX}(\omega)$ *and* $S_{YY}(\omega)$ *are the* input and output power spectral density and H(ω) is the transfer function then CO6- U

(a) $S_{XX}(\tau) = |H(\omega)|^2 S_{YY}(\omega)$ (b) $S_{XY}(\tau) = |H(\omega)|^2 S_{XX}(\omega)$

(c) $S_{YY}(\omega) = |H(\omega)|^2 S_{XX}(\omega)$ (d) None of the above

9. Large sample size is ______ CO6- U

(a) 30 (b) >30 (c) <30 (d) None of the above

10. Small sample size is ______ CO6- U

(a) 30 (b) >30 (c) <30 (d)None of the above

PART – B (5 x 2= 10Marks)

11. Find the mean for the discrete RV X with probability distribution CO1-App

X	-2	-1	0	1
P(X)	0.4	0.1	0.2	0.3

12. The joint PDF of the RV (X,Y) is given by CO2-Ana

$$f(x,y) = \begin{cases} e^{-(x+y)}, & 0 < x, y < \infty \\ 0 & , otherwise \end{cases}$$

Are X And Y Independent?

13. Compute the auto correlation function $|R_{xx}(\tau)| \le R_{xx}(0)$ CO3-App

14. Calculate the value of the system transfer function, if the input of the system with impulse response h(t) = $e^{-\alpha t}$ U(t). CO4-App

15. A sample of size 10 has mean 58, standard deviation18.4 and population mean 50, Compute the calculated value of 't' distribution. CO5- Ana

PART – C (5 x 16= 80 Marks)

16. (a) (i) A Random Variable X has the following probability distribution CO1 -App (8)

X=x	0	1	2	3	4	5	6	7
P(X= x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2$+ k

Find (i) 'k'

(ii) P(X > 6), P(0< X < 4)

(ii) Define Binomial distribution. Find the moment generating function and Hence find mean and variance CO1-App (8)

Or

(b) If the density function of a continuous random variable X is given by CO1 -App (16)

$$f(x) = \begin{cases} ax & ; \ 0 \le x \le 1 \\ a & ; 1 \le x \le 2 \\ 3a - ax & ; \ 2 \le x \le 3 \\ 0 & otherwise \end{cases}$$

(i) Find the value of "a"

(ii) Find the distribution function of X

17. (a) If joint probability distribution function X and Y is given by P(x, y) = k (2x + 3y), for x = 0, 1, 2 & y= 1, 2,3 then find all marginal and conditional probability distribution function of X and Y and
also find P(X+Y). CO2 -Ana (16)

Or

(b) From the following data, find CO2- Ana (16)

(i) the two regression equations

(ii) the coefficient of correlation between the marks in Economics and Statistics

(iii) the most likely marks in Statistics when marks in Economics are 30

Marks in Economics	25	28	35	32	31	36	29	38	34	32
Marks in Statistics	43	46	49	41	36	32	31	30	33	39

18. (a) (i) If the auto correlation function of the random binary transmission is given by $R_{XX}(\tau) = \begin{cases} 1 - \dfrac{|\tau|}{T} & ; |\tau| \le T \\ 0 & ; |\tau| \ge T \end{cases}$ Find the Power spectral density function CO3- App (8)

(ii) A stationary process has an autocorrelation function given by $R(\tau) = 25 + \dfrac{4}{1 + 6\tau^2}$ Find the Mean and Variance CO3- App (8)

Or

(b) (i) If the Power spectral density of a WSS processes is given by CO3- App (8)

$$\mathbf{S(\omega) = \begin{cases} \dfrac{b}{a}\left(a - |\omega|\right) & ; \quad |\omega| \le a \\ 0 & ; \quad |\omega| > a \end{cases}}$$

Find the auto correlation function of the Process.

(ii) Find the power spectral densities of the following auto correlation function $R(\tau) = e^{\frac{-\alpha^2\tau^2}{2}}$ CO3- App (8)

19. (a) A random process X (t) having the autocorrelation function CO4- App (16)

$R_{XX}(\tau) = P e^{-\alpha|\tau|}$ Where b is a constant is applied to the input of the system with impulse response h(t)= $e^{-bt}U(t)$ where b is a constant. Find the autocorrelation of the output Y (t).

Or

(b) If X (t) is a WSS process and if CO4- App (16)

$$Y(t) = \int_{-\infty}^{\infty} h(u)X(t-u)du \text{ then}$$

(i). $R_{XY}(\tau) = R_{XX}(\tau) * h(\tau)$ **(ii).** $R_{YY}(\tau) = R_{XY}(\tau) * h(-\tau)$

(iii). $S_{XY}(\omega) = S_{XX}(\omega) * H(\omega)$ **(iv).** $S_{YY}(\omega) = S_{XX}(\omega) * |H(\omega)|^2$

20. (a) Two researchers A and B adopted different techniques while rating the student's level. Identify the Sampling distribution; Can you say that the techniques adopted by them are significant? CO5- Ana (16)

Researchers	Below Average	Average	Above Average	Genius	Total
A	40	33	25	2	100
B	86	60	44	10	200
Total	126	93	69	12	300

OR

(b) Two independent samples of sizes 9 and 7 from a normal population had the following values of the variables. CO5- Ana (16)

Sample I	18	13	12	15	12	14	16	14	15
Sample II	16	19	13	16	18	13	15		

Identify the sampling distribution, Do the estimates of the population variance differ significantly.