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Question Paper Code: R2M11

B.E./B.Tech. DEGREE EXAMINATION, MAY 2024

Second Semester

Artificial Intelligence and Machine Learning

R21UMA211-FOURIER SERIES, PARTIAL DIFFERENTIAL EQUATIONS AND

COMPLEX ANALYSIS

(Regulations R2021)

Duration: Three hours

Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The term $(a_1 \cos x + b_1 \sin x)$ in Fourier series is called ----- CO6- U
(a) First harmonic (b) Second harmonic (c) Third harmonic (d) Fourier Coefficients
2. The Fourier constant b_n in $(-\pi, \pi)$ for $x \sin x$ is _____ CO1-App
(a) x^2 (b) $3x$ (c) 0 (d) 1
3. The elimination of one arbitrary function will result in a PDE of ---- order CO2-App
(a) first (b) second (c) third (d) fourth
4. The complete solution of $z = px + qy + pq$ is ----- CO2-App
(a) $z = ax + by + ab$ (b) $z = ax - by - ab$ (c) $z = ax + by - ab$ (d) $z = ax - by + ab$
5. An insulated rod of length 60 cm has its ends at A and B kept at CO3-App
 20°C and 80°C respectively, then its steady state solution is
(a) $x-20$ (b) $4x+20$ (c) $x+20$ (d) $x+60$
6. $2u_{xx} + 3u_x u_y + u_{yy} = x$ is a type of CO3-App
(a) Hyperbolic (b) Parabolic (c) elliptic (d) cyclic

7. The function $f(z) = \frac{1}{z^2+4}$ is not analytic at $z =$ _____ CO6- U
 (a) 2 (b) -2 (c) 2i (d) $\pm 2i$
8. The mapping $w = z^2$ is not conformal at _____ CO6- U
 (a) 0 (b) -1 (c) 1 (d) 2
9. The value of $\int_C \frac{dz}{z+2}$, $C: |z| = 1$ is _____ CO6- U
 (a) $2\pi i$ (b) $-2\pi i$ (c) $4\pi i$ (d) 0
10. The residue of $f(z) = \frac{4}{z^3(z-2)}$ at its simple pole is _____ CO5- App
 (a) $\frac{4}{7}$ (b) $\frac{1}{2}$ (c) $\frac{1}{7}$ (d) $\frac{3}{4}$

PART – B (5 x 2= 10 Marks)

11. Find a_0 and a_n in the Fourier series of $f(x) = x$ in $(0, 2\pi)$ CO1- App
12. Solve $\sqrt{p} + \sqrt{q} = 1$ CO2- App
13. The ends A and B of a rod of length 10cm long have their temperature kept at 20°C & 70°C . Find the steady state temperature distribution on the rod. CO3-App
14. Show that the function $e^x \sin y$ is harmonic CO4- App
15. Evaluate $\int_C \frac{e^{-z}}{z+1} dz$ where C is $|z| = \frac{1}{2}$ using Cauchy integral formula. CO5- App

PART – C (5 x 16= 80Marks)

16. (a) (i) Express $f(x) = \frac{1}{2}(\pi - x)$ as a Fourier series of period 2π in the interval $0 < x < 2\pi$. CO1- App (10)
- (ii) Find the Half range cosine series for $f(x) = x$ in $(0, \pi)$ CO1- App (6)

Or

- (b) CO1-App (16)

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y:	1.8	0.3	0.5	2.16	1.3	1.76	1.8

The table of values of the function $y = f(x)$ is given below
 Find a Fourier series up to the third harmonic to represent $f(x)$ in terms of x

17. (a) (i) Solve $(D^2 - 5DD' + 6D'^2)z = e^{x+y} + \sin(x - y)$ CO2-App (8)
(ii) Solve $x(y - z)p + y(z - x)q = z(x - y)$ CO2-App (8)
Or
- (b) (i) Solve $Z = px + qy + p^2 - q^2$ CO2-App (8)
(ii) Form a PDE by eliminating arbitrary functions from $\varphi(x^2 + y^2 + z^2, x + y + z) = 0$. CO2-App (8)
18. (a) A String is stretched and fastened to two points 1 apart. Motion is started by displacing the string into the form $y = K(lx - x^2)$ from which it is released at $t = 0$. Find the displacement of any point at a distance 'x' at any time 't' CO3 -App (16)
Or
- (b) A bar of 30cm long with insulated sides has its ends A and B kept at 20°C and 80°C respectively. Until steady state condition prevails. The temperature at A is then suddenly raised to 60°C and at the same instant B is lowered to 40°C and maintained thereafter. Find the subsequent temperature distribution in the bar. CO3- App (16)
19. (a) (i) Using Milne Thomson method, find the Analytic function CO4-App (8)
given that $u = \frac{\sin 2x}{\cosh 2y - \cos 2x}$
- (ii) Find the image of $|z - 1| = 1$ under the transformation $w = \frac{1}{z}$ CO4-App (8)
Or
- (b) (i) Find the bilinear transformation from $-1, 0, 1$ to $0, i, 3i$ CO4-App (8)
- (ii) If $f(z) = u + iv$ is a regular function of z in a domain D the following relation hold in D . $\nabla^2 |f(z)|^2 = 4 |f'(z)|^2$ CO4-App (8)

20. (a) (i) Evaluate CO5-App (8)

$$f(z) = \int_C \frac{\cos \pi z^2 + \sin \pi z^2}{(z+1)(z+2)} dz \text{ by using Cauchy's Integral}$$

formula where C is $|z| = 3$

(ii) Expand $\frac{z-1}{(z+2)(z+3)}$ as Laurent's series valid in the CO5-App (8)

region $2 < |z| < 3$.

Or

(b) Using Contour integration, to prove CO5-App (16)

$$\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + a^2)(x^2 + b^2)} dx = \frac{\pi}{a+b} \quad a > b > 0$$