A   **Reg. No. :**

B.E./B.Tech. DEGREE EXAMINATION, MAY 2024

Second Semester

Artificial Intelligence and Machine Learning

21UMA211-FOURIER SERIES, PARTIAL DIFFERENTIAL EQUATIONS AND

COMPLEX ANALYSIS

(Regulations 2021)

Duration: Three hours   Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The constant term in the Fourier expansion for $f(x) = k$, in $(0, 2\pi)$ is   CO6- U

    (a) 2k   (b) 3k   (c) k   (d) 0

2. The root mean square value of $f(x)$ in $(0, l)$ is ---------   CO1- App

    (a) l   (b) 1/2   (c) $l/\sqrt{3}$ (d) 2l

3. The PDE obtained from $z = (x+a)(y+b)$ is __.   CO2-App

    (a) $3z = px + qy$ (b) $py - qx = 0$ (c) $z = pq$ (d) $px+qy = 0$

4. The general solution of $(D^2 - 8DD' +12D'^2) z = 0$ is _____   CO2-App

    (a) $f_1 (y + 2x) + f_2 (y+6 x)$ (b $f_1 (y + 2x) + f_2 (y-6x)$

    (c) $z = xf(y+3x/2) - g(y+3x/2)$ (d) $z = xf(y-3x/2)$

5. Classify the equation $u_{xx}+u_{yy} = 0$ is _____   CO6- U

    (a) parabolic   (b)hyperbolic   (c) elliptic   (d) cyclic

6. An insulated rod of length 60 cm has its ends at A and B kept at 20^oC CO3-App
    and 80^oC respectively, then its steady state solution is

    (a) x-20   (b)4x+20   (c) x+20   (d) x+60

7. The critical point of the transformation $w = z + \frac{1}{z}$ are ____   CO4- App

    a) ±1 b) ±2 a) ±1 b) ±2

8. Find the fixed points of $f(z) = \dfrac{1}{z-2i}$ CO4-App

    (a) i   (b) 2i   (c) 3i   (d) 0

9. The residue of f(z) = $\dfrac{4}{z^3(z-2)}$ at its simple pole is _____ CO5-App

    (a) $\dfrac{4}{7}$ (b) $\dfrac{3}{4}$ (c) $\dfrac{1}{7}$ (d) $\dfrac{3}{4}$

10. The value of $\int_C \dfrac{dz}{z+2}$, $c : |z| = 1$ is _____ CO6-U

    (a) $2\pi i$ (b) 0   (c) $4\pi i$ (d) 0

PART – B (5 x 2= 10Marks)

11. Give the expression for the Fourier series coefficient bn for the function f(x) = x defined in $(-\pi, \pi)$ CO1–App

12. Find the complete solution of $z = px + qy + p^2 - q^2$ CO2–App

13. The ends A and B of a rod of length 10cm long have their temperature kept at $20\degree c$ & $70\degree c$. Find the steady state temperature distribution on the rod. CO3-App

14. Prove that analytic function with the constant real part is constant CO4-App

15. Evaluate $\int_c \dfrac{z}{z-2} dz$ where C is $|z| = 2$ CO5–App

PART – C (5 x 16= 80Marks)

16. (a) (i) Express $f(x) = x^2$ as a Fourier series of period 2π in the interval $0 < x < 2\pi$. CO1- App   (8)

    (ii) Find the Half range sine series for $f(x) = x$ in $(0, \pi)$ CO1-App   (8)

<div align="center">Or</div>

(b) The table of values of the function $y = f(x)$ is given below: CO1- App   (16)

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y:	1.0	1.4	1.9	1.7	1.5	1.2	1.0

Find a Fourier series upto the third harmonic to represent f(x) in terms of x

17. (a) (i) Solve $(D^2 - 5DD' + 6D'^2)z = e^{x+y} + sin(x-y)$ CO2 -App   (8)

    (ii) Solve $x(y-z)p + y(z-x)q = z(x-y)$ CO2 -App   (8)

(b) (i) Solve $Z = px + qy + p^2 - q^2$ CO2 -App   (8)

(ii) Form a PDE by eliminating arbitrary functions from   CO2 -App   (8)

$\varphi(x^2 + y^2 + z^2, x + y + z) = 0..$

18. (a) A String is stretched and fastened to two points l apart .Motion is   CO3-App   (16)
started by displacing the Velocity $\lambda (lx-x^2)$ from which it is
released at t=0.Find the displacement of any point at a distance
'x' at any time 't'.

Or

(b) A bar of 10cm long with insulated sides has its ends A and B kept   CO3-App   (16)
at 50° c and 100^oc respectively. Until steady state condition
prevails. The temperature at A is then suddenly raised to 90^oc and
at the same instant B is lower to 60^oc and maintained thereafter.
Find the subsequent temperature distribution in the bar.

19. (a) (i) Using Milne Thomson method, find the Analytic function   CO4-App   (8)

given that $u = \dfrac{\sin 2x}{\cosh 2y - \cos 2x}$

(ii) Find the bilinear transformation from -1,0,1 to 0,i,3i   CO4-App   (8)

Or

(b) (i) Find the image of $|z-1| = 1$ under the transformation   CO4-App   (8)

$w = \dfrac{1}{z}$

(ii) If f (z) is analytic whose real part is constant must itself be a   CO4-App   (8)
constant

20. (a) (i) Evaluate using Cauchy's Integral formula for   CO5-App   (8)

$f(z) = \int\limits_{C} \dfrac{2z-1}{(z+1)(z-3)} dz$, where 'C' : $|z| = 2.$

(ii) Evaluate $f(z) = \dfrac{1}{(z+1)(z+3)}$ in Laurent series valid for the   CO5-App   (8)

region $1 < |z| < 3.$

Or

(b) Using contour integration, to find the value of   CO5-App   (16)

$\int\limits_{-\infty}^{\infty} \dfrac{1}{(x^2 + a^2)(x^2 + b^2)} dx$

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