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Question Paper Code: U5M01

B.E./B.Tech. DEGREE EXAMINATION, APRIL 2024

Fifth Semester

Artificial Intelligence & Data Science

21UMA501 - LINEAR ALGEBRA

(Regulations 2021)

Duration: Three hours

Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The $\dim(\mathbb{R}^3)$ is _____ CO6- U
(a) 1 (b) 2 (c) 3 (d) 0
2. The trivial subspaces of a vector space V are _____ CO6-U
(a) $\{0\}$ (b) V (c) W (d) $\{0\}$ and V
3. If $T: V \rightarrow W$ be linear transformation then $T(0) =$ _____ CO6- U
(a) 0 (b) 1 (c) 2 (d) 3
4. In a linear transformation $T: V \rightarrow W$ the kernel of T is a subspace of _____ CO6- U
(a) V (b) W (c) both V and W (d) none of these
5. $\langle x, x \rangle = 0$ if and only if _____ CO6- U
(a) $x = 1$ (b) $x \neq 1$ (c) $x = 0$ (d) $x \neq 0$
6. In a vector space V , if $\langle x, y \rangle = \langle y, z \rangle$ then _____ CO6- U
(a) $y = z$ (b) $y \neq z$ (c) $y = -z$ (d) none of these
7. The Hermitian Matrices containing some _____ numbers. CO6- U
(a) Real (b) Imaginary (c) Natural (d) None of these
8. All Eigen values of a _____ matrix are real CO6- U
(a) Symmetric (b) Skew Symmetric (c) Hermitian (d) None of these

9. A square matrix A is said to be ----- if the determinant value of A is zero. CO6- U
- (a) Singular (b) Non - Singular (c) Symmetric (d) none of these
10. Linear Programming deals with the ----- of a function of decision variable. CO6- U
- (a) Optimization (b) Formulation (c) Technique (d) none of these

PART – B (5 x 2= 10Marks)

11. Prove that $W = \{(a,0,0) | a \in \mathbb{R}\}$ is a subspace of \mathbb{R}^3 CO1-App
12. Define Rank nullity theorem CO6-U
13. Find the norm of $(1,2,3)$ in $V_3(\mathbb{R})$ with standard inner product. CO3 -App
14. Let $A = \begin{pmatrix} 11 & -6i \\ 4i & 1 \end{pmatrix}$ Find the Characteristic equation of A CO6-U
15. State whether $y = \tan x$ can be expanded as a Fourier series. CO5- App

PART – C (5 x 16= 80Marks)

16. (a) (i) Verify the vectors $(1,2,0)$, $(2,3,0)$, $(8,13,0)$ in \mathbb{R}^3 is a basis of \mathbb{R}^3 CO1- App (8)
(ii) In \mathbb{R}^3 determine whether $(1,7,-4)$ is expressed as a linear combination of $(1,-3,2)$ and $(2,-1,1)$ CO1- App (8)
- Or
- (b) (i) Find the dimension of the subspace spanned by the vectors $(1,2,-3)$, $(0,0,1)$, $(-1,2,1)$ in $V_3(\mathbb{R})$ CO1 -App (8)
(ii) Write the Vector $V = (1,-2,5)$ as a Linear combination of the Vectors $e_1 = (1,1,1)$, $e_2 = (1,2,3)$ and $e_3 = (2,-1,1)$ in \mathbb{R}^3 CO1 -App (8)
17. (a) (i) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be linear transformation defined by $T(a_1, a_2) = (a_1 + a_2, a_1)$ then find nullity(T), rank(T), Is T one-to-one? Is T onto? Also check the rank nullity theorem. CO2- App (8)
(ii) Find the matrix of the linear transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $T(a,b) = (2a-3b, a+b)$ for the standard basis of \mathbb{R}^2 CO2- App (8)
- Or
- (b) $T: V_1(\mathbb{R}) \rightarrow V_3(\mathbb{R})$ is defined by $T(a_1) = (a_1, 2a_1^2, a_1^3)$ Verify whether T is a linear transformation CO2- App (16)

18. (a) (i) Show that the following function defines an inner product on $V_2(R)$ where $x = (x_1, x_2)$ and $y = (y_1, y_2)$ and $\langle x, y \rangle = 6x_1y_1 + 7x_2y_2$ CO3- App (8)
- (ii) If $x = (2, 1 + i, i)$ and $y = (2 - i, 2, 1 + 2i)$ then verify Schwarz's inequality. CO3- App (8)

Or

- (b) Apply Gram-Schmidt process to construct an orthonormal basis for $V_3(R)$ with the standard inner product for the basis $\{v_1, v_2, v_3\}$ where $v_1 = (1, -1, 0)$, $v_2 = (2, -1, -2)$ and $v_3 = (1, -1, 2)$ CO3- App (16)

19. (a) Construct a QR factorization for a matrix CO4- App (16)

$$A = \begin{pmatrix} -4 & 4 & 2 \\ 4 & -4 & 1 \\ 2 & 1 & 0 \end{pmatrix}$$

Or

- (b) Determine the Cholesky Decomposition for CO4- App (16)

$$A = \begin{pmatrix} 16 & -3 & 5 & -8 \\ -3 & 16 & -5 & -8 \\ 5 & -5 & 24 & 0 \\ -8 & -8 & 0 & 21 \end{pmatrix}$$

20. (a) From the following information state whether the condition of the child is associated with the condition of the house CO5- App (16)

Condition of child	Condition of house		Total
	Clean	Dirty	
Clean	69	51	120
Fairly clean	81	20	101
Dirty	35	44	79
Total	185	115	300

Or

(b) The Savings bank account of a customer showed an average balance of Rs. 150 and a SD of Rs 50 assuming that the account balances are normally distributed. CO5- App (16)

i) What percentage of account is over Rs. 200 ?

ii) What percentage of account is between Rs.120 and Rs. 170 ?

iii) What percentage of account is less than Rs. 75 ?

($0 < z < 0.4 = 0.3811$, $0 < z < 0.6 = 0.2257$,

$0 < z < 1.5 = 0.4332$, $0 < z < 1 = 0.3143$)