A

Reg. No. :

B.E./B.Tech. DEGREE EXAMINATION, MAY 2024

Third Semester

Agriculture Engineering

21UMA326- TRANSFORM TECHNIQUES AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to Biomedical and Biotechnology Engineering)

(Regulations 2021)

Duration: Three hours   Maximum: 100 Marks

Answer All Questions

PART A - (10x 1 = 10 Marks)

1. The term ($a_1\cos x + b_1\sin x$) in fourier series is called _____   CO6-U

   (a) First harmonic   (b) Second harmonic   (c) Third harmonic   (d) Fourier Coefficients

2. Cos x is a periodic function with period _____   CO6- U

   (a) π (b) 2π (c)$\pi/3$ (d) $2\pi/3$

3. If $F[f(x)] = f(s)$ then the function is said to be _____   CO6- U

   (a) Odd   (b) Even   (c) Self-Reciprocal   (d) Periodic

4. If $F[f(x)] = f(s)$ thenF[f(ax)] = _____   CO2- App

   (a) $\dfrac{1}{-a}F\left(\dfrac{s}{a}\right)$ (b)$\dfrac{1}{a}F\left(\dfrac{s}{a}\right)$ (c) $\dfrac{1}{|a|}F\left(\dfrac{s}{a}\right)$ (d) $\dfrac{1}{s}F\left(\dfrac{s}{a}\right)$

5. If Z{f(t)} = F(Z), then Z{e^{-at}f(t)} = _____   CO6- U

   (a)F [e^{aT}]   (b)F [Ze^{aT}]   (c)F [Ze^{-aT}]   (d)F [e^{-aT}]

6. Z [n f(n)] = _____   CO6- U

   (a)$-z\dfrac{d}{dz}F(Z)$ (b)$z\dfrac{d}{dz}F(Z)$ (c)$\dfrac{d}{dz}F(Z)$ (d)$z^n\dfrac{d}{dz}F(Z)$

7. The PDE obtained from z = (x+a)(y+b) is __.   CO4- App

   (a) 3z = px + qy   (b) py - qx = 0   (c) z = pq   (d) px+qy = 0

8. The particular integral of $(D^2 - 4DD' + 3D'^2)\, z = e^{x+y}$ is _____   CO4- App

    (a) $\dfrac{xe^{x+y}}{2}$ (b) $-\dfrac{xe^{x+y}}{2}$ (iii) $\dfrac{x^2 e^{x+y}}{2}$ (iv) $\dfrac{-x^2 e^{x+y}}{2}$

9. Classify the equation y2uxx+uyy = 0 is _____   CO6- U

    (a) parabolic   (b) hyperbolic   (c) elliptic   (d) cyclic

10. An insulated rod of length 60 cm has its ends at A and B kept at 20oC and   CO5- App
80oC respectively, then its steady state solution is

    (a) x-20   (b) 4x+20   (c) x+20   (d) x+60

<p align="center">PART – B (5 x 2= 10Marks)</p>

11. Find $a_0\ and\ a_n$ in the Fourier series of $f(x) = x + x^3$ in (-π,π)   CO1- App

12. Find the Fourier cosine transform of $f(x) = \dfrac{1}{1+x^2}$ CO2- App

13. Find $Z\left[\sin\left(\dfrac{n\pi}{2}\right)\right]$ CO3- App

14. Solve: $\left(D^2 - 4DD' + 4D'^2\right)Z = \sin(x + y).$ CO4- App

15. Write the three Possible solutions of the one dimensional wave equations   CO5- U

<p align="center">PART – C (5 x 16= 80Marks)</p>

16. (a) (i) Find the Fourier series of $f(x) = \begin{cases} -1 + x, & -\pi < x < 0 \\ 1 + x, & 0 < x < \pi \end{cases}$ of   CO1 -App   (8)

periodicity 2π.

(ii) Find the Half range cosine series for $f(x) = x(\pi - x)$ in $(0, \pi)$.   CO1 -App   (8)

Deduce that $\dfrac{1}{1^4} + \dfrac{1}{2^4} + \dfrac{1}{3^4}\ldots\ldots = \dfrac{\pi^4}{90}$

<p align="center">Or</p>

(b) The table of values of the function $y = f(x)$ is given below: CO1 -App   (16)

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y:	1.0	1.4	1.9	1.7	1.5	1.2	1.0

Find a Fourier series upto the third harmonic for $f(x)$ in terms of x.

17. (a) Find the Fourier transform of $f(x) = \begin{cases} a - |x|, & \text{if} \quad |x| \le a \\ 0 & \text{if} \quad |x| > a \end{cases}$ and   CO2 -App   (16)

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hence deduce that $(i) \int_0^\infty \left(\dfrac{\sin t}{t} \right)^2 dt$ (ii) $\int_0^\infty \left(\dfrac{\sin t}{t} \right)^4 dt$

Or

(b) Evaluate (i) $\int_0^\infty \dfrac{x^2 \, dx}{\left(x^2 + a^2\right)\left(x^2 + b^2\right)}$ (ii) Evaluate $\int_0^\infty \dfrac{dx}{\left(x^2 + 49\right)^2}$ CO2 -App   (16)

18. (a)

(i) Solve the difference equation $y_{n+2} + 6 y_{n+1} + 9 y_n = 2^n$ CO3- App   (8)

given that $y_0 = 0,\ y_1 = 0$

(ii) Using Convolution theorem find $Z^{-1}\left[\dfrac{8 z^2}{(4z - 3)(2z + 1)} \right]$ CO3- App   (8)

Or

(b)

(i) Solve the difference equation $y_{n+2} + 4 y_{n+1} + 3 y_n = 2^n$ CO3- App   (8)

given that $y_0 = 0,\ y_1 = 0$

(ii) Evaluate $Z[r^n \cos n\theta]\ and\ Z[r^n \sin n\theta]$.   CO3- App   (8)

19. (a) (i) Solve $x\left(z^2 - y^2\right)p + y\left(x^2 - z^2\right)q = z\left(y^2 - x^2\right)$ CO4-App   (8)

(ii) Solve $\left(D^2 - DD' - 2D'^2 \right)z = 2x + 3y + e^{3x+4y}$ CO4-App   (8)

Or

(b) (i) Solve $x\left(y - z\right)p + y\left(z - x\right)q = z\left(x - y\right)$ CO4-App   (8)

(ii) Solve $\left(D^2 - 7DD' + 6D'^2 \right)z = e^{2x+y}$ CO4-App   (8)

20. (a) A String is stretched and fastened to two points l apart .Motion is started by displacing the string into the form y=K(lx-x^2) from which it is released at t=0.Find the displacement of any point at a distance 'x' at any time 't'.   CO5- App   (16)

(b) A tightly String with fixed end points x=0 and x=l is initially at rest in its equilibrium position. If its set vibrating giving each point at velocity
λ(lx-x^2).Find the displacement .   CO5- App   (16)