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Reg. No. : | | | | | | | | | | | | |

Question Paper Code: U2M07

B.E./B.Tech. DEGREE EXAMINATION, MAY 2024

Second Semester

Agriculture Engineering

21UMA207- CALCULUS COMPLEX ANALYSIS AND TRANSFORM TECHNIQUES

(Regulations 2021)

(Common to bio medical and biotechnology engineering branches)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The Particular integral of $y'' + 4y' + 4y = 0$ is ______ CO1-App

   (a) xe^{-2x} (b) xe^{2x} (c) $x^2 e^{2x}$ (d) 0

2. $\frac{1}{D^2}(\cos x)$ = _____ CO6-R

   (a) sin x (b) - cos x (c) cos x (d) tan x

3. Divergence of vector $x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ at (1, 2, -3) is ______ CO2-App

   (a) 8 (b) 4 (c) -3 (d) 0

4. If $\varphi = x^2 + y^2 - z - 10$ then $|\nabla \varphi|$ at (1, 1, 1) is ______ CO2-App

   (a) $2(\vec{i} + \vec{j} + \vec{k})$ (b) $2\vec{i} + 2\vec{j} - \vec{k}$ (c) 3 (d) 9

5. The critical point of the transformation $w = z + \frac{1}{z}$ are _____ CO6- U

   (a) ± 1 (b) ± 2 (c) $\pm i$ (d) $-i$

6. The mapping $w = z^2$ is not conformal at CO6- R

   (a) 0 (b) -1 (c) 1 (d) 2

7. Simple pole is a pole of order ________ CO6-U

   (a) 1 (a) 2 (a) 3 (a) 4

8. The poles of z cot z is _______ CO6-U

(a) 0 (b) $\pm n\pi$ (c) 1 (d) π

9. L (sin h at) = _____ CO6-R

(a) $\frac{s}{s^2 - a^2}$ (b) $\frac{a}{s^2 - a^2}$ (c) $\frac{s}{s^2 + a^2}$ (d) $\frac{a}{s^2 + a^2}$

10. L [t f(t)] = _____ CO6-R

(a) F'(s) (b)–F'(s) (c)F(s) (d) –F(s)

PART – B (5 x 2= 10Marks)

11. Solve $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$ CO1-App

12. Find the Directional derivative of $\varphi = 4xz^2 + x^2yz$ at (1,-2,-1) in the direction $2\vec{i} + 3\vec{j} + 4\vec{k}$. CO2-App

13. Find the fixed point of $w = \frac{2z-5}{z+4}$ CO3-App

14. Evaluate $\int_c \frac{e^{-z}}{z+1} dz$ where C is $|z| = \frac{1}{2}$ using Cauchy integral formula CO4-App

15. Estimate L[t cost] CO5-App

PART – C (5 x 16= 80Marks)

16. (a) (i) Using method of variation of parameters solve $(D^2 + a^2)y = \tan ax$. CO1-App (8)

(ii) At the start of an experiment, there are 100 bacteria. If the bacteria follow an exponential growth pattern with rate k = 0.02. What will be the population after 5 hours? How long will it take for the population to double? CO1- App (8)

Or

(b) (i) Solve: $(x^2D^2 + xD + 1)y = x \sin(\log x)$ CO1- App (8)

(ii) Solve: $(D^2 - 4D + 3)y = \sin 3x + e^{2x}$ CO1- App (8)

17. (a) Verify Divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - xz)\vec{j} + (z^2 - xy)\vec{k}$ over the rectangular parallelepiped x = 0, x = a, y = 0, y = b, z = 0, z = c. CO2-App (16)

Or

(b) (i) Using Green's theorem, Evaluate $\int_C (3x^2 - 8y^2)\,dx + (4y - 6xy)dy$ where C is the boundary of the region defined by X = 0, Y = 0, X + Y= 1 in the XY plane. CO2 -App (8)

(ii) Prove that $\bar{F} = (x^2 + xy^2)\bar{\imath} + (y^2 + x^2y)\bar{\jmath}$ is irrotational vector and compute the Scalar potential such that $\bar{F} = \nabla\emptyset$. CO2 -App (8)

18. (a) (i) Using Milne Thomson method, find the Analytic function given that $u = \dfrac{\sin 2x}{\cosh 2y - \cos 2x}$ CO3-App (8)

(ii) Find the bilinear transformation from -1,0,1 to 0,i,3i CO3-App (8)

Or

(b) (i) Find the image of $|z-1| = 1$ under the transformation $w = \dfrac{1}{z}$ CO3-App (8)

(ii) If f(z) is analytic whose real part is constant must itself be a constant. CO3-App (8)

19. (a) (i) Evaluate $f(z) = \int_C \dfrac{\cos \pi z^2 + \sin \pi z^2}{(z-1)(z-2)}dz$ by using Cauchy's Integral formula where C is $|z| = 3$ CO4-App (8)

(ii) Expand $\dfrac{z-1}{(z+2)(z+3)}$ as Laurent's series valid in the region $2 < |z| < 3$ CO4-App (8)

Or

(b) Using Contour integration, to prove CO4-App (16)

$$\int_{-\infty}^{\infty} \frac{x^2}{(x^2+a^2)(x^2+b^2)}dx = \frac{\pi}{a+b} \quad a > b > 0$$

20. (a) (i) Solve the differential equation $\dfrac{d^2y}{dt^2} + y = \sin 2t$; $y(0) = 0$; $y'(0) = 0$ by using Laplace transform method. CO5-App (8)

(ii) Find the inverse Laplace Transform of $\dfrac{s+3}{(s+1)(s^2+2s+3)}$ CO5-App (8)

Or

(b) (i) Find the Laplace transform of f(t) = CO5-App (8)

$$f(t)=\begin{cases} k, & 0 \le t \le a \\ -k, & a \le t \le 2a \end{cases}$$

(ii) Solve by using convolution theorem CO5-App (8)

$$L^{-1}\left[\frac{1}{(s+1)(s^2+1)}\right]$$