Reg. No. :

Question Paper Code : 95383

5 Year M.Sc. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015

Second Semester

Software Engineering

XCS 122/10677 SW 202 — ANALYTICAL GEOMETRY AND REAL AND COMPLEX ANALYSIS

(Common to M.Sc. Information Technology and Computer Technology)

(Regulations 2003/2007/2010)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Evaluate $\iint xy\,dx\,dy$ over the positive quadrant of $x^2+y^2=1$

2. Evaluate : $\int_0^1\int_0^2\int_0^3 xyz\;dzdydx$.

3. Find 'a' such that $\overline{F}=(ax+3y)\bar{i}+(2y-3z)\bar{j}+(x-3z)\bar{k}$ is solenoidal.

4. State "Stoke's theorem".

5. Find the intercepts made by the plane $2x+3y+6z-12=0$ on the co-ordinate axes.

6. Find the equation to the tangent plane at $(1,-1,2)$ to the sphere $x^2+y^2+z^2-2x+4y+6z-12=0$.

7. Prove that $f(z)=e^z$ is analytic everywhere in the complex plane.

8. Prove that xy^2 cannot be real part of an analytic function.

9. Evaluate $\int_C (z-2)^n dz$.

10. State Cauchy Residue theorem.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Change the order of integralion in $\int_0^1\int_0^x dydx$. and hence evaluate. (8)

(ii) Evaluate $\iiint_v dxdydz$, where the V is the volume of the tetrahedron whose vertices are (0,0,0), (0,1,0), (1,0,0) and (0,0,1). (8)

Or

(b) (i) Find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ by double integration. (8)

(ii) Find the volume of the sphere $x^2+y^2+z^2=1$ by triple integrals. (8)

12. (a) (i) Using Green's theorem, evaluate $\int_C (\sin x - y)dx - \cos x\, dy$ where C is the triangle with vertices (0,0), $\left(\frac{\pi}{2},0\right)$ and $\left(\frac{\pi}{2},1\right)$. (6)

(ii) Show that $\overline{F}=\left(2xy+z^3\right)\overline{i}+x^2\overline{j}+3xz^2\overline{k}$ is a conservative force field. Find the scalar potential and the work done by $\overline{F}$ in moving an object in this field from (1,-2,1) to (3,1,4). (10)

Or

(b) (i) What is the angle between the surfaces $x^2+y^2+z^2=9$ and $x^2+y^2-z-3=0$ at (2,-1,2)? (6)

(ii) Verify Gauss Divergence theorem for $\overline{F}=4xz\overline{i}-y^2\overline{j}+yz\overline{k}$ over the cube $x=0, x=1, y=0, y=1, z=0, z=1$. (10)

13. (a) (i) Find the foot of the perpendicular drawn from $p(-2,7,-1)$ to the plane $2x-y+z=0$ and also the image of p in the plane. (8)

(ii) Show that the lines $\frac{x-4}{5}=\frac{y-3}{-2}=\frac{z-2}{-6}$ and $\frac{x-3}{4}=\frac{y-2}{-3}=\frac{z-1}{-7}$ are coplanar. Find their point of intersection and equation of the plane in which they lie. (8)

Or

(b) (i) Find the equation of the sphere which passes through the points (1,–4,3), (1,–5,2) and has its centre on the line $\frac{x+4}{-4}=\frac{y+2}{1}=\frac{z-6}{3}$. (8)

(ii) Find the equation of the tangent planes to the sphere $x^2+y^2+z^2-4x+2y-6z+5=0$ which are parallel to the plane $2x-2y-z=0$. (8)

14. (a) (i) Show that $f(z) = z^n$ is analytic for positive integral values of n and find $f'(z)$. (8)

(ii) Find the analytic function whose imaginary part is $e^x(x\sin y + y\cos y)$. (8)

Or

(b) (i) If $P + iQ$ is an analytic function of z and if $P = \frac{2\sin 2x}{e^{2y} + e^{-2y} - 2\cos 2x}$ determine Q. (8)

(ii) If $f(z) = u + w$ is an analytic function of $z = x + iy$ prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. (8)

15. (a) (i) Evaluate $\int_0^{2\pi} \frac{\sin^2\theta}{5 - 3\cos\theta} d\theta$, using contour integration. (10)

(ii) Find the Laurent's series of $f(z) = \frac{1}{z(1-z)}$ valid in the region $|z+1| < 1$. (6)

Or

(b) (i) If $f(a) = \int_C \frac{3z^2 + 7z + 1}{z - a} dz$, where C is the cirele $|z| = 2$, find the values of $f(3), f'(1-i)$ and $f''(1-i)$. (10)

(ii) Evaluate $\int_C \frac{\cos\pi z^2 + \sin\pi z^2}{(z+1)(z+2)} dz$, where C is $|z| = 3$, using Cauchy's residue theorem. (6)
