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Question Paper Code : 95279

5 Year M.Sc. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

Second Semester

Software Engineering

EMA 002 — ANALYTICAL GEOMETRY AND REAL AND COMPLEX ANALYSIS

(Common to 5 Year M.Sc. Software Systems)

(Regulations 2010)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy(x+y)\, dy dx$.

2. Change the order of integration in $\int_0^2 \int_x^2 (x^2+y^2)\, dx\, dy$.

3. Find the function ϕ, if $\text{grad}\,\phi = (y^2 - 2\,xyz^3)\,i + (3 + 2xy - x^2 z^3)\,j + (6z^3 - 3x^2yz^2)k$.

4. Show that $F = (x+2y)\,i + (y+3z)j + (x-2z)k$ is solenoidal.

5. Find the equation of the plane through the point $(-1,2,-3)$ and perpendicular to the line joining the points $(-3,2,4)$ and $(5,4,1)$.

6. Find the values of k, if the line $\frac{x-2}{3} = \frac{y-1}{2} = \frac{z-3}{k}$ and $\frac{x-3}{k} = \frac{y-2}{3} = \frac{z-4}{5}$ are coplanar.

7. Find the value of a,b,c,d so that the function $f(z) = (x^2 + axy + by^2) + i(cx^2 + d\,xy + y^2)$ may be analytic.

8. Verify whether the function $u = e^y \cosh x$ is harmonic?

9. Evaluate using Cauchy's integral formula $\int_C \frac{z+2}{z} dz$, where C is the semicircle $|z| = 2$ in the upper half of the z-plane.

10. Find the residue of $f(z) = \frac{z+2}{(z-2)(z+1)^2}$ at $z = 2$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Evaluate $\int_0^1 \int_0^{1-z} \int_0^{y-z} xyz \, dx \, dy \, dz$. (8)

(ii) Change the order of integration and hence evaluate $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy \, dy \, dx$. (8)

Or

(b) (i) Evaluate $\iint (x+y) \, dx \, dy$ over the region in the positive quadrant bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (8)

(ii) Evaluate $\iiint_V (x+y+z) \, dx \, dy \, dz$ where V is the volume of the rectangular parallelopiped bounded by $x = 0$, $x = a$, $y = 0$, $y = b$, $z = 0$ and $z = c$. (8)

12. (a) (i) Show that $F = (y^2 + 2xz^2) i + (2xy - z) j + (2x^2 z - y + 2z) k$ is irrotational and hence find its scalar potential ϕ. (6)

(ii) Verify the divergence theorem when $F = x^2 i + y^2 j + z^2 k$, where S is the surface of the cuboid formed by the planes $x = 0$, $x = a$, $y = 0$, $y = b$, $z = 0$, $z = c$. (10)

Or

(b) (i) If $r = |r|$, where r is the position vector of the point (x, y, z) with respect to the origin, find $\nabla f(r)$ and $\nabla^2 f(r)$. (8)

(ii) Verify Stoke's theorem for $F = (y - z + 2) i + (yz + 4) j - xz k$ and S is the open surface of the cube formed by $x = 0$, $x = 2$, $y = 0$, $y = 2$, $z = 0$ and $z = 2$. (8)

13. (a) (i) Find the equation of the planes through the line of intersection of the planes $x+3y+6=0$ and $3x-y-4z=0$, whose distance from the origin is unity. (8)

(ii) Find the length of shortest distance between the pairs of lines $\frac{x-1}{1}=\frac{y-2}{-2}=\frac{z-3}{3}$ and $\frac{x+1}{2}=\frac{y}{-1}=\frac{z-1}{3}$. (8)

Or

(b) (i) Find the equation of the sphere passing through the points $(1,1,-2)$ and $(-1,1,2)$ and having its centre on the line $x+y-z=1$, $2x-y+z=2$. (8)

(ii) Find the equation of the plane which passes through the point $(1,2,-1)$ and which contains the line $\frac{x+1}{2}=\frac{y-1}{3}=\frac{z+2}{-1}$. (8)

14. (a) (i) Prove that $u=e^{x}(x\sin y+y\cos y)$ is harmonic. Find the corresponding analytic function $w=u+iv$. (8)

(ii) If $f(z)$ is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=|f'(z)|^2$. (8)

Or

(b) (i) Discuss the conformal mapping $w=z^2$, along with figures. (8)

(ii) Find the bilinear transformation which maps the points $z=-i$, 0, i into $w=-1,i,1$ respectively. (8)

15. (a) (i) Evaluate $\int_C \frac{z+4}{z^2+2z+5}dz$, where $C:|z+1+i|=2$, using Cauchy's integral formula. (8)

(ii) Find the residues of $f(z)=\frac{z^2}{(z-1)(z+2)^2}$, at its isolated singularities using Laurent's series expansion. (8)

Or

(b) (i) Evaluate $\int_0^{2\pi}\frac{\sin^2\theta}{5-3\cos\theta}d\theta$ by using contour integration. (8)

(ii) Evaluate $\int_0^{\infty}\frac{x^2}{(x^2+1)(x^2+4)}dx$ by using contour integration. (8)