Reg. No. [ ]

**Question Paper Code : 95282**

5 Year M.Sc. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

Fourth Semester

Software Engineering

EMA 005 — DISCRETE MATHEMATICS

(Regulations 2010)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Define disjunctive and conjunctive normal forms of a statement.
2. Negate the statement "Every student in this class is intelligent" in two different ways.
3. Give an example of a relation that is neither symmetric nor antisymmetric.
4. Find the inverse of the function $f : R \to R^+$ defined by $f(x) = e^{2x-5}$.
5. State the basic properties of a group.
6. What is group code?
7. Prove that the additive inverse of every element of the ring is unique.
8. What is the prime field of C?
9. Write the distributive inequalities of a Lattice.
10. Simplify $(a \cdot b)' + (a + b)'$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent. (8)

(ii) Use mathematical induction to show that $1 + 2 + 2^2 + 2^3 + \ldots + 2^n = 2^{n-1} - 1$ for all nonnegative integers $n$. (8)

Or

(b) (i) Show that $t \wedge s$ can be derived from the premises $p \to q, q \to \sim r, r, p \vee (t \wedge s)$. (8)

(ii) Without constructing the truth tables, find the principal disjunctive normal form of the statement $(\sim p \to q) \wedge (q \leftrightarrow p)$. (8)

12. (a) (i) Let $R$ be the relation on the set of ordered pairs of positive integers such that $(a,b),(c,d) \in R$ if and only if $a+d=b+c$. Show that $R$ is an equivalence relation. (8)

(ii) If $A = \{x \in R / x \neq 1/2\}$ and $f: A \to R$ is defined by $f(x) = 4x/(2x-1)$, show that $f$ is invertible and find the range of $f$. (8)

Or

(b) (i) Let $R$ be the relation consisting of all pairs $(x,y)$ such that $x$ and $y$ are strings of uppercase and lowercase English letters with the property that for every positive integer $n$, the $n^{th}$ characters in $x$ and $y$ are the same letter, either uppercase or lowercase. Show that $R$ is an equivalence relation. (8)

(ii) Show that the functions $f: R \to A$ and $g: A \to R$, where $A = (1,\infty)$ defined by $f(x) = 3^{2x} + 1$ and $g(x) = \frac{1}{2}\log_3(x-1)$ are inverses. (8)

13. (a) (i) If $G$ is the set of all ordered pairs $(a,b)$, where $a(\neq 0)$ and $b$ are real and the binary operation $*$ on $G$ is defined by $(a,b)*(c,d) = ac, bc+d)$, Show that $(G,*)$ is an non-ableian group. Show also that the subset $H$ of all those elements of $G$ which are of the form $(a,b)$ is a subgroup of $G$. (8)

(ii) If $C^*$ and $R^*$ are multiplication groups of non-zero complex numbers and non-zero real numbers respectively and if the mapping $f: C^* \to R^*$ is defined by $f(z) = |z|$. Show that the mapping $f$ is a homomorphism. (8)

Or

(b) (i) State and prove Lagrange's theorem. (8)

(ii) Find the code words generated by the encoding function $e: B^2 \to B^5$ with respect to the parity check matrix $H = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. (8)

14. (a) (i) Show that a set $M$ of all $2 \times 2$ matrices over integers forms a ring under a matrix multiplication and matrix addition. (8)

(ii) Show that the set $R[x]$ of all polynomials over an arbitrary ring $R$ is a ring with respect to addition and multiplication of polynomials. (8)

Or

(b) (i) Show that the set of numbers of the form $a+b\sqrt{2}$ with $a$ and $b$ as rational numbers is a field. (8)

(ii) Prove that the set of integers $R=\{0,1,2,3,4\}$ forms a field under addition modulo 5 and multiplication modulo 5. (8)

15. (a) (i) Consider a set $S=\{a,b,c\}$. Is the relation of set inclusion $\subseteq$ is a partial order on $P(S)$ where $P(S)$ is a power set of S? (8)

(ii) Prove that the complement of every element on a Boolean algebra $B$ is unique. (8)

Or

(b) (i) If $(L,\leq)$ is a Lattice, prove that for any $a,b,c \in L, a \wedge (b \vee c) = (a \wedge b) \vee c$. (8)

(ii) In a Boolean algebra, prove that the statements $a+b=b$, $a.b=a$ are equivalent. (8)